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MATHEMATICS

Trajectory Attractor for a System of Two Reaction-Diffusion Equations with Diffusion Coefficient $\delta(t) \to 0+$ as $t \to +\infty$[1]

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The method of trajectory attractors (see [1–6]) makes it possible to effectively study the limit behavior of solutions to partial differential equations for which the theory of global attractors (see [7, 8]) is not directly applicable, for example, due to nonunique solvability of the corresponding Cauchy problem. In the present paper, we construct the trajectory attractor for a nonautonomous reaction-diffusion system where one equation has a time-dependent diffusion coefficient that tends to zero as $t \to +\infty$. In [9, 10], an analogous problem has been studied for autonomous reaction-diffusion systems having one time-independent diffusion coefficient that approaches zero as a parameter of the problem.

1. REACTION-DIFFUSION SYSTEM WITH A DIFFUSION COEFFICIENT $\delta(t)$ THAT APPROACHES ZERO AS $t \to +\infty$

In a bounded domain $\Omega \Subset \mathbb{R}^3$, we consider the following nonautonomous reaction-diffusion system:

$$\partial_t u = \Delta u - f(u, v) + g_1(x), \qquad (1)$$

$$\partial_t v = \delta(t)\Delta v - h(u, v) + g_2(x), \qquad (2)$$

where $u = u(x, t)$, $v = v(x, t)$, $x \in \Omega$, $t \geq 0$. The Laplace operator Δ acts in the x-variable of the domain Ω. At the boundary $\partial\Omega$, we set the homogeneous Dirichlet conditions

$$u|_{\partial\Omega} = 0, \quad v|_{\partial\Omega} = 0. \qquad (3)$$

In Eq. (2), the diffusion coefficient δ depends on time t. We assume that $\delta(\cdot) \in L_\infty^{\mathrm{loc}}(\mathbb{R}_+)$, $\delta(t) \geq 0$ for $t \geq 0$, and

$$\int_t^{t+1} \delta(s)ds \to 0 \quad \text{as} \quad t \to +\infty. \qquad (4)$$

The nonlinear functions $f, h: \mathbb{R}^2 \to \mathbb{R}$ are continuous and satisfy the following inequalities:

$$\sigma_1(|u|^{p_1} + |v|^{p_2}) - c \leq f(u, v)u + h(u, v)v$$
$$\leq C(|u|^{p_1} + |v|^{p_2} + 1), \qquad (5)$$

$$|f(u, v)|^{p_1/(p_1-1)} + |h(u, v)|^{p_2/(p_2-1)}$$
$$\leq C(|u|^{p_1} + |v|^{p_2} + 1), \quad \forall u, v \in \mathbb{R} \qquad (6)$$

for some positive constants σ_1, c, C, and $p_1, p_2 \geq 2$. We also assume that $h \in C^1(\mathbb{R}^2)$, $h(0, 0) = 0$, and the following inequalities hold:

$$0 < \sigma_2 \leq \frac{\partial h}{\partial v}(u, v), \quad \left|\frac{\partial h}{\partial u}(u, v)\right| \leq D, \quad \forall u, v \in \mathbb{R}. \qquad (7)$$

The positive quantities σ_1 and σ_2 in (5) and (7) reflect the dissipation of the system (1)–(2). We set $\sigma := \min\{\sigma_1, \sigma_2\}$.

Note that we do not assume that the nonlinear function f in Eq. (1) is differentiable and so the Cauchy problem for system (1)–(3) can have more than one solution in the corresponding function space.

The functions $g_1(x)$ and $g_2(x)$ in Eqs. (1) and (2) belong to the spaces

$$g_1 \in L_2(\Omega), \quad g_2 \in H_0^1(\Omega).$$

An example of system (1), (2) is a nonautonomous analog of the FitzHugh–Nagumo system:

$$\partial_t u = \Delta u - u(u-\beta)(u-1) - v,$$

$$\partial_t v = \delta(t)\Delta v + \alpha u - \gamma v,$$

where $f(u, v) = u(u - \beta)(u - 1) + v$, $h(u, v) = \gamma v - \alpha u$, and $\alpha, \beta, \gamma \geq 0$. For this example, $\sigma = \min\{1, \gamma\}$, $p_1 = 4$, $p_2 = 2$, and $g_1 \equiv 0$, $g_2 \equiv 0$ (see [7, 11, 12]).

For simplicity of notations, we set $H := L_2(\Omega)$, $V := H_0^1(\Omega)$ and denote the norms in these spaces: $\|u\| := \|u\|_H$, $\|u\|_1 := \|u\|_V$.

For arbitrary functions $u(\cdot) \in L_{p_1}(0, M; L_{p_1}(\Omega))$ and $v(\cdot) \in L_{p_2}(0, M; L_{p_2}(\Omega))$, it follows from (6) that $f(u, v) \in L_{q_1}(0, M; L_{q_1}(\Omega))$ and $h(u, v) \in L_{q_2}(0, M; L_{q_2}(\Omega))$, where $q_i = \dfrac{p_i}{p_i - 1}$, $i = 1, 2$, and

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$$\|f(u, v)\|^{q_1}_{L_{q_1}(0, M; L_{q_1})} + \|h(u, v)\|^{q_2}_{L_{q_2}(0, M; L_{q_2})}$$

$$\le C_1(\|u\|^{p_1}_{L_{p_1}(0, M; L_{p_1})} + \|v\|^{p_2}_{L_{p_2}(0, M; L_{p_2})} + 1).$$

Definition 1. A pair of functions $(u(x, t), v(x, t))$, $x \in \Omega$, $t \ge 0$, is called a *weak solution* to system (1)–(3) if for every $M > 0$,

$$u \in L_{p_1}(0, M; L_{p_1}(\Omega)) \cap L_2(0, M; V),$$

$$v \in L_{p_2}(0, M; L_{p_2}(\Omega)) \cap L_2(0, M; V),$$

and the functions $u(x, t)$ and $v(x, t)$ satisfy Eqs. (1) and (2) in the sense of distributions with values in the spaces $H^{-r_1}(\Omega)$ and $H^{-r_2}(\Omega)$, respectively, where $r_i = \max\left\{1, 3\left(\frac{1}{2} - \frac{1}{p_i}\right)\right\}$, $i = 1, 2$ (see [1, 8, 13]). The exponents r_i are chosen such that Eqs. (1), (2) and the Sobolev embedding theorem implies

$$\partial_t u(\cdot) \in L_{q_1}(0, M; H^{-r_1}(\Omega)),$$

$$\partial_t v(\cdot) \in L_{q_2}(0, M; H^{-r_2}(\Omega)).$$

If $(u(\cdot), v(\cdot))$ is a weak solution to (1), (2), then it follows from these equations that

$$u(\cdot) \in L_\infty(0, M; H), \quad v(\cdot) \in L_\infty(0, M; H).$$

In addition, using the Lions–Magenes lemma (see [14]), we obtain

$$u(\cdot) \in C_w([0, M]; H),$$

$$v(\cdot) \in C_w([0, M]; H), \quad \forall M > 0.$$

Consequently, for every $t \ge 0$, the values of functions $u(\cdot, t)$ and $v(\cdot, t)$ are well defined as elements of the space H, and, in particular, the following initial conditions make sense:

$$u|_{t=0} = u_0 \in H, \quad v|_{t=0} = v_0 \in H. \tag{8}$$

In [1], we proved that for any weak solution $(u(\cdot), v(\cdot))$ to system (1), (2), the functions $u(\cdot)$ and $v(\cdot)$ belong to $C(\mathbb{R}_+; H)$, while the function $\|u(t)\|^2 + \|v(t)\|^2$ is absolutely continuous for $t \ge 0$ and satisfies the energy identity

$$\frac{1}{2}\frac{d}{dt}\{\|u(t)\|^2 + \|v(t)\|^2\} + \|\nabla u(t)\|^2 + \delta(t)\|\nabla v(t)\|^2$$

$$+ \int_\Omega f(u(x, t), v(x, t))u(x, t)dx$$

$$+ \int_\Omega h(u(x, t), v(x, t))v(x, t)dx$$

$$= \int_\Omega \{g_1(x)u(x, t) + g_2(x)v(x, t)\}dx.$$

Formally, to obtain this identity, we multiply Eq. (1) by $u(x, t)$ and Eq. (2) by $v(x, t)$. Then we integrate over $x \in \Omega$ and sum the results.

Proposition 1. *For any weak solution* $(u(\cdot), v(\cdot))$ *to the system* (1), (2) *with initial data* (8), *the following inequalities hold*

$$\|u(t)\|^2 + \|v(t)\|^2 + 2\int_0^t (\|u(s)\|_1^2 + \delta(s)\|v(s)\|_1^2)e^{-\sigma(t-s)}ds$$

$$\le (\|u_0\|^2 + \|v_0\|^2)e^{-\sigma t} + R_1^2, \tag{9}$$

$$2\int_t^{t+1} (\|u(s)\|_1^2 + \delta(s)\|v(s)\|_1^2)ds$$

$$+ \sigma\int_t^{t+1} (\|u(s)\|^{p_1}_{L_{p_1}} + \|v(s)\|^{p_2}_{L_{p_2}})ds$$

$$\le (\|u_0\|^2 + \|v_0\|^2)e^{-\sigma t} + R_2^2, \quad \forall t \ge 0; \tag{10}$$

the quantities R_1 *and* R_2 *depend on* σ, $\|g_1\|$, *and* $\|g_2\|$.

We now assume that

$$v|_{t=0} = v_0 \in V. \tag{11}$$

Under this assumption, the existence of a weak solution to the problem (1)–(3), (8) is established using the Galerkin method.

Proposition 2. *Under assumptions* (11), *the problem* (1)–(3), (8) *has a weak solution* $(u(\cdot), v(\cdot))$ *such that* $v(\cdot) \in L_\infty(\mathbb{R}_+; V)$, *and the following inequality holds*:

$$\|v(t)\|_1^2 \le \|v_0\|_1^2 e^{-\sigma t} + C_2(\|u_0\|^2 + \|v_0\|^2)e^{-\sigma t} + R^2, \quad \forall t \ge 0, \tag{12}$$

where $R = R(\sigma, D, \|g_2\|, R_1)$, $C_2 = C_2(\sigma, D)$.

The Lions–Magenes lemma implies that $v(\cdot) \in C_w(\mathbb{R}_+; V)$; i.e., the values $v(\cdot, t) \in V$ are well-defined for all $t \ge 0$.

We now define a linear space $\mathcal{F}_+^{\mathrm{loc}}$. By definition,

$$\mathcal{F}_+^{\mathrm{loc}} = \left\{\begin{array}{c} (y(x, t), z(x, t)),\ x \in \Omega,\ t \ge 0 \,| \\ y \in L_\infty^{\mathrm{loc}}(\mathbb{R}_+; H) \cap L_2^{\mathrm{loc}}(\mathbb{R}_+; V) \cap L_{p_1}^{\mathrm{loc}}(\mathbb{R}_+; L_{p_1}(\Omega)), \\ z \in L_\infty^{\mathrm{loc}}(\mathbb{R}_+; V) \cap L_{p_2}^{\mathrm{loc}}(\mathbb{R}_+; L_{p_2}(\Omega)), \\ \partial_t y \in L_{q_1}^{\mathrm{loc}}(\mathbb{R}_+; H^{-r_1}(\Omega)),\ \partial_t z \in L_{q_2}^{\mathrm{loc}}(\mathbb{R}_+; H^{-r_2}(\Omega)) \end{array}\right\}. \tag{13}$$

We consider the linear subspace $\mathcal{F}_+^b \subset \mathcal{F}_+^{\rm loc}$ of functions (y, z) with a finite norm:

$$\|(y, z)\|_{\mathcal{F}_+^b} := \|y\|_{L_\infty(\mathbb{R}_+; H)} + \|y\|_{L_2^b(\mathbb{R}_+; V)} + \|y\|_{L_{p_1}^b(\mathbb{R}_+; L_{p_1})} + \|\partial_t y\|_{L_{q_1}^b\left(\mathbb{R}_+; H^{-r_1}\right)} + \|z\|_{L_\infty(\mathbb{R}_+; V)} + \|z\|_{L_{p_2}^b(\mathbb{R}_+; L_{p_2})} + \|\partial_t z\|_{L_{q_2}^b\left(\mathbb{R}_+; H^{-r_2}\right)}. \tag{14}$$

Recall that the norm in the space $L_p^b(\mathbb{R}_+; X)$ is defined by the formula $\|y\|_{L_p^b(\mathbb{R}_+; X)} := \sup_{t \ge 0} \int_t^{t+1} \|y(s)\|_X^p ds$. The space $\mathcal{F}_+^b$ with norm (14) is a Banach space.

We now define the space $\mathcal{K}_+^\delta(N)$ of weak solutions (trajectories) to system (1), (2) with diffusion coefficient δ, which depends on $N > 0$.

Definition 2. The space $\mathcal{K}_+^\delta(N)$ consists of functions $(u(\cdot), v(\cdot)) \in \mathcal{F}_+^{\rm loc}$ such that (1) the pair $(u(t), v(t))$, $t \ge 0$ is a weak solution to (1), (2); (2) the function $v(\cdot)$ satisfies the inequality

$$\|v(t)\|_1^2 \le Ne^{-\sigma t} + R^2, \quad \forall t \ge 0, \tag{15}$$

with σ and R from (12).

The space $\mathcal{K}_+^\delta(N)$ is nonempty. Indeed, we take a weak solution to (1), (2) with initial data $u_0 \in H$ and $v_0 \in V$, which was constructed in Proposition 2. If the norms $\|u_0\|$, $\|v_0\|_1$ satisfy inequality $\|v_0\|_1^2 + C_2(\|u_0\|^2 + \|v_0\|_2) \le N$, then, due to (12), this weak solution belongs to $\mathcal{K}_+^\delta(N)$.

Consider the family of the translation operators $T(\tau)$, $\tau \ge 0$ acting on the space $\mathcal{F}_+^{\rm loc}$ by the formula

$$T(\tau)(y(t), z(t)) = (y(t+\tau), z(t+\tau)), \quad t \ge 0. \tag{16}$$

Proposition 3. *The space of trajectories $\mathcal{K}_+^\delta(N)$ lies in $\mathcal{F}_+^b$, and the following inequality holds*:

$$\|T(\tau)(u, v)\|_{\mathcal{F}_+^b} \le C_3(\|u(0)\|^2 + N)e^{-\rho\tau} + R_3^2, \quad \forall \tau \ge 0. \tag{17}$$

2. THE LIMIT REACTION-DIFFUSION SYSTEM WITH ZERO DIFFUSION COEFFICIENT AND ITS TRAJECTORY ATTRACTOR

Consider the system that is “limit” with respect to (1), (2) as the diffusion coefficient $\delta \equiv 0$:

$$\partial_t u = \Delta u - f(u, v) + g_1(x), \tag{18}$$

$$\partial_t v = -h(u, v) + g_2(x). \tag{19}$$

This system is autonomous. At the boundary $\partial\Omega$, we set the conditions

$$u|_{\partial\Omega} = 0, \quad v|_{\partial\Omega} = 0. \tag{20}$$

By a *weak solution* to (18)–(20), we mean a pair of functions $(u(x, t), v(x, t))$,

$$u(\cdot) \in L_{p_1}^{\rm loc}(\mathbb{R}_+; L_{p_1}(\Omega)) \cap L_2^{\rm loc}(\mathbb{R}_+; V),$$

$$v(\cdot) \in L_{p_2}^{\rm loc}(\mathbb{R}_+; L_{p_2}(\Omega)) \cap L_\infty^{\rm loc}(\mathbb{R}_+; V),$$

that satisfy Eqs. (18), (19) in the sense of the theory of distributions with values in the spaces $H^{-r_1}(\Omega)$ and $H^{-r_2}(\Omega)$, respectively, (the exponents r_1 and r_2 have been defined in 1). For $t = 0$, we set the initial conditions

$$u|_{t=0} = u_0 \in H, \quad v|_{t=0} = v_0 \in V. \tag{21}$$

Propositions 1 and 2 are applicable to the system (18), (19) since it is a particular case of system (1), (2) for $\delta \equiv 0$. Therefore, the reduced inequalities (9), (10) are valid, where we remove terms containing the function $\delta(s)$. In addition, Cauchy problem (18)–(21) has a weak solution (u, v) that satisfies (15). From Eqs. (18), (19) we obtain

$$\partial_t u(\cdot) \in L_{q_1}(0, M; H^{-r_1}(\Omega)),$$

$$\partial_t v(\cdot) \in L_{q_2}(0, M; L_{q_2}(\Omega)).$$

Note that $L_{q_2}(\Omega) \subset H^{-r_2}(\Omega)$ and, therefore, the constructed weak solutions belong to the space $\mathcal{F}_+^{\rm loc}$ (see (13)).

Similarly to the space $\mathcal{K}_+^\delta(N)$, we define the space of trajectories $\mathcal{K}_+^0(N)$ for system (18), (19), which consists of its weak solutions $(u(\cdot), v(\cdot))$ such that the function $v(\cdot)$ satisfies inequality (15). As it was noticed before, the space $\mathcal{K}_+^0(N)$ is nonempty and Proposition 3 holds: namely, $\mathcal{K}_+^0(N)$ belongs to $\mathcal{F}_+^b$, and inequality (17) is true.

We now construct the trajectory attractor for system (18), (19).

In the space $\mathcal{F}_+^{\rm loc}$ (see (13)), we consider the following topology, which we define in terms of the weak convergence of the corresponding sequences from $\mathcal{F}_+^{\rm loc}$. By definition, the sequence of functions $\{(y_m(\cdot), z_m(\cdot))\} \subset \mathcal{F}_+^{\rm loc}$ converges as $m \to \infty$ to $(y(\cdot), z(\cdot)) \in \mathcal{F}_+^{\rm loc}$ in the topology $\Theta_+^{\rm loc}$ if, for each $M > 0$, the following convergences hold as $m \to \infty$:

(1) $y_m(\cdot) \rightharpoonup y(\cdot)$ weak-$*$ in $L_\infty(0, M; H)$, weakly in $L_2(0, M; V)$, and weakly in $L_{p_1}(0, M; L_{p_1}(\Omega))$;

(2) $z_m(\cdot) \rightharpoonup z(\cdot)$ weak-$*$ in $L_\infty(0, M; V)$ and weakly in $L_{p_2}(0, M; L_{p_2}(\cdot))$;

(3) $\partial_t y_m(\cdot) \rightharpoonup \partial_t y(\cdot)$ weakly in $L_{q_1}(0, M; H^{-r_1}(\cdot))$;

(4) $\partial_t z_m(\cdot) \rightharpoonup \partial_t z(\cdot)$ weakly in $L_{q_2}(0, M; H^{-r_2}(\cdot))$.

Note that the space $\mathcal{F}_+^{\mathrm{loc}}$ equipped with topology Θ_+^{loc} is a linear Hausdorff space. In addition, this space is a Frechét–Urysohn space with a countable topology base (see, for example, [1]).

Remark 1. Any ball of radius r

$$\mathcal{B}_r = \{\|(y, z)\|_{\mathcal{F}_+^b} \le r\}$$

in the space $\mathcal{F}_+^b$ is a compact set in the topology Θ_+^{loc}. The set $\mathcal{B}_r$ itself equipped with topology Θ_+^{loc} is a compact metric space (see, for example, [15]).

Consider the translation operators $T(\tau)$, $\tau \ge 0$, acting on $\mathcal{F}_+^{\mathrm{loc}}$ by the formula (16). These operators form a semigroup $\{T(\tau)\} := \{T(\tau), \tau \ge 0\}$ on $\mathcal{F}_+^{\mathrm{loc}}$ called the *translation semigroup*. Note that $\{T(\tau)\}$ takes the space of trajectories $\mathcal{K}_+^0(N)$ to itself, since the system (18), (19) is autonomous; that is,

$$T(\tau)\colon \mathcal{K}_+^0(N) \to \mathcal{K}_+^0(N), \quad \forall \tau \ge 0.$$

To construct the trajectory attractor, the key step is the following result proved in [10].

Proposition 4. *The space* $\mathcal{K}_+^0(N)$ *is closed in* Θ_+^{loc} *for every* $N \ge 0$.

Recall that the set $P \subseteq \mathcal{K}_+^0(N)$ is *called absorbing* for the semigroup $\{T(\tau)\}$ if, for every set $B \subset \mathcal{K}_+(N)$ bounded in $\mathcal{F}_+^b$, there is $\tau_1 = \tau_1(B) \ge 0$ such that $T(\tau)B \subseteq P$ for all $\tau \ge \tau_1$. The set $P \subseteq \mathcal{K}_+^0(N)$ is called *attracting* for the semigroup $\{T(\tau)\}$ in the topology Θ_+^{loc} if every neighbourhood $\mathbb{O}(P)$ of the set P in the topology Θ_+^{loc} is an absorbing set.

We now define the trajectory attractor for the semigroup $\{T(\tau)\}|_{\mathcal{K}_+^0(N)}$.

Definition 3. The set $\mathfrak{A} \subset \mathcal{K}_+^0(N)$ is called the *trajectory attractor* for the semigroup $\{T(\tau)\}$ on $\mathcal{K}_+^0(N)$ if $\mathfrak{A}$ is bounded in the norm $\mathcal{F}_+^b$, compact in the topology Θ_+^{loc}, strictly invariant with respect to $\{T(\tau)\}$, $T(\tau)\mathfrak{A} = \mathfrak{A}$, for all $\forall \tau \ge 0$, and $\mathfrak{A}$ is an attracting set for $\{T(\tau)\}$ on $\mathcal{K}_+(N)$ in the topology Θ_+^{loc}.

We briefly describe the construction of the trajectory attractor. It follows from inequality (17) that the set

$$P_0 = \left\{(u, v) \in \mathcal{K}_+^0(N) \,|\, \|(u, v)\|_{\mathcal{F}_+^b} \le 2R_3^2\right\}$$

is an absorbing set for the semigroup $\{T(\tau)\}$ on $\mathcal{K}_+^0(N)$ and, in addition, $T(\tau)P_0 \subseteq P_0$ for all $\tau \ge 0$. As it was noticed before, the set P_0 with topology Θ_+^{loc}, being a ball in $\mathcal{F}_+^b$, is a compact metric space. It is easy to verify that the semigroup $\{T(\tau)\}$ is continuous in the topology Θ_+^{loc}. Consequently, we have a continuous semigroup $\{T(\tau)\}$ acting on a compact metric space P_0. Then the general theorem on the existence of a global attractor is applicable (see, e.g., [7, 8]). The global attractor $\mathfrak{A} \subseteq P$ of the semigroup $\{T(\tau)\}|_{P_0}$ is constructed by the formula

$$\mathfrak{A} := \bigcap_{\tau \ge 0}\left[\bigcup_{\theta \ge \tau} T(\theta)P_0\right]_{\Theta_+^{\mathrm{loc}}}.$$

The set $\mathfrak{A}$ is clearly bounded in $\mathcal{F}_+^b$, compact in Θ_+^{loc}, strictly invariant, that is, $T(\tau)\mathfrak{A} = \mathfrak{A}$ for all $\tau \ge 0$, and it follows easily that $\mathfrak{A}$ attracts any bounded in $\mathcal{F}_+^b$ set $B \subseteq \mathcal{K}_+^0(N)$ in the topology Θ_+^{loc}. Therefore, $\mathfrak{A}$ is the trajectory attractor of $\{T(\tau)\}$ on $\mathcal{K}_+^0(N)$.

Proposition 5. *The trajectory attractor* $\mathfrak{A} = \mathfrak{A}(N)$ *constructed above in the space* $\mathcal{K}_+^0(N)$ *is independent of* N. *In particular,* $\mathfrak{A} = \mathfrak{A}(0)$; *that is,*

$$\sup\{\|v(t)\|_1^2 \,|\, t \ge 0\} \le R^2, \quad \forall (u, v) \in \mathfrak{A}.$$

3. CONVERGENCE OF SOLUTIONS OF THE ORIGINAL REACTION-DIFFUSION SYSTEM (1), (2) TO THE TRAJECTORY ATTRACTOR $\mathfrak{A}$ OF THE LIMIT SYSTEM (18), (19).

We now formulate the main theorem of the paper.

Theorem 1. *For any* $N > 0$ *and for every set of trajectories* $B = \{(u(t), v(t))\} \in \mathcal{K}_+^\delta(N)$ *of the nonautonomous system* (1), (2), *which is bounded in the space* $\mathcal{F}_+^b$, *the family of translations* $T(\tau)B = \{(u(t+\tau), v(t+\tau))\}$ *converges in the topology* Θ_+^{loc} as $\tau \to +\infty$ *to the trajectory attractor* $\mathfrak{A}$ *of the limit autonomous system* (18), (19):

$$T(\tau)B \to \mathfrak{A}(\tau \to +\infty) \ \text{ in } \ \Theta_+^{\mathrm{loc}}.$$

In conclusion, we summarize the main result of the paper: the nonautonomous reaction-diffusion system (1), (2) with the diffusion coefficient $\delta(t)$ that vanishes as $t \to +\infty$ (in the sense (4)) has a trajectory attractor that coincides with the trajectory attractor $\mathfrak{A}$ of the corresponding limit autonomous reaction-diffusion system (18), (19) with zero diffusion coefficient, $\delta \equiv 0$.

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