Logic Colloquium

European Summer Meeting of the Association for Symbolic Logic

BOOK OF ABSTRACTS

Adam Mickiewicz University in Poznań

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Logic Colloquium 2021
book of abstracts

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About

The Logic Colloquium is the European Summer Meeting of the Association for Symbolic Logic, that in 2021 will be held from 19th to 24th of July at the Adam Mickiewicz University in Poznań, Poland. It is organized jointly by the AMU Faculties: of Psychology and Cognitive Science and of Mathematics and Computer Science. This year, it is an on-line event. However, we are not going to miss this perfect opportunity to present to all of the prospective participants our University and the beautiful city of Poznań: please have a look at the 'Venue' and 'Local Guide' sections of the website, and come visit us in more travel-friendly time!

Committees

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- Wojciech Buszkowski, Adam Mickiewicz University in Poznań
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- Kazimierz Swirydowicz, Adam Mickiewicz University
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Contributed talks

All contributed talks in alphabetical order

1. Claudio Agostini and Eugenio Colla, An algebraic characterization of Ramsey Monoids
2. Aleksi Anttila, Maria Aloni and Fan Yang, A logic for modelling free choice inference
3. Aizhan Altayeva, Beibut Kulpeshov and Sergey Sudoplatov, On algebras of binary formulas for almost ω-categorical weakly o-minimal theories
4. Pavel Arazim, Logic not being serious
5. John Baldwin, Finer classification of Strongly minimal sets
6. Nikolay Bazhenov, Dariusz Kalociński and Michał Wrocławski, Degree spectra of unary recursive functions on naturals with standard ordering
7. Gaia Belardinelli and Rasmus K. Rendsvig, Epistemic Planning with Attention as a Bounded Resource
8. Dylan Bellier, Massimo Benerecetti, Dario Della Monica and Fabio Mogavero, Good-for-Game QPTL: An Alternating Hodges Semantics
9. Bruno Bentzen, How do we intuit mathematical constructions?
10. Nick Bezhanishvili and Fan Yang, Intermediate logics in the team semantics setting
11. Laurent Bienvenu, Valentino Delle Rose and Tomasz Steifer, Computable randomness relative to almost all oracles
12. Katalin Bimbo, Abaci running backward
13. Nicola Bonatti, Two questions concerning quantifiers rules
14. Marta Fiori Carones, Measuring the strength of Ramsey-theoretic statements over $RCA0^*$
15. Matteo de Ceglie, The V-logic Multiverse and MAXIMIZE
16. Yong Cheng, The interpretation degree structure of r.e. theories for which the first incompleteness theorem holds
17. Horatiu Cheval, General metatheorems in proof mining
18. Anahit Chubaryan and Arsen Hambardzumyan, On non-monotonous properties of some propositional proof systems

19. Anahit Chubaryan, Sargis Hovhannisyan and Hayk Gasparyan, Comparison of two propositional proof systems by lines and by sizes
20. Gabriel Ciobanu, Various notions of infinity for finitely supported structures
21. Vittorio Cipriani, Cantor-Bendixson theorem in the Weihrauch lattice
22. Willem Conradie and Valentin Goranko, Algorithmic correspondence for relevance logics
23. Ludovica Conti, Logicality and Abstraction
24. Andrés Cordón-Franco, F. Félix Lara-Martín and Manuel J.S. Loureiro, On determinacy of Lipschitz and Wadge games in second order arithmetic
25. Jakub Dakowski, Aleksandra Draszewska, Barbara Adamska, Dominika Juszczak, Łukasz Abramowicz and Robert Szymański, Addressing logic students' proof making difficulties with Plugin Oriented Programming and gamification
26. Karol Duda and Aleksander Iwanow, A finitely presented group with undecidable amenability
27. Dmitry Emelyanov, Beibut Kulpeshov and Sergey Sudoplatov, On algebras of binary formulas for partially ordered theories
28. Christian Espindola, Categoricity theorems in infinite quantifier languages
29. Andrzej Gajda, Abductive reasoning in a neural-symbolic system
30. Francesco Gallinaro, Around exponential algebraic closedness
31. Luke Gardiner, Countable Exponent Partition Relations on the Real Line
32. Margarita Gaskova, Boolean algebras autostable relative to n-constructivizations
33. Azza Gaysin, H-Coloring Dichotomy in Proof Complexity
34. Meghdad Ghari, A temporal logic of justification and obligation
35. Michał Tomasz Godziszewski and Luca San Mauro, Quotient structures, philosophy of computability theory and computational structuralism
36. Michał Tomasz Godziszewski, Fairness and Jutsified Representation in Judgment Aggregation and Belief Merging
37. Lew Gordeev and Edward Haeusler, On Proof Theory in Computational Complexity

38. Davit Harutyunyan, On Some Associative Formula with Functional Variables

39. Taneli Huuskonen, Cromulence logic: duty meets preference

40. Martina Iannella, The complexity of convex bi-embeddability among countable linear orders

41. Raheleh Jalali, On hard theorems for substructural logics

42. Josiah Jacobsen-Grocott, A Characterization of the Strongly η-Representable Many-One Degrees

43. Marcin Jukiewicz and Dorota Leszczyńska-Jasion, Improving the work of a genetic algorithm in proof-search tasks

44. Vladimir Kanovei, On the 'Definability of definable' problem of Alfred Tarski

45. Ruaan Kellerman and Valentin Goranko, Approximating trees as coloured linear orders and complete axiomatisations of some classes of trees

46. Mohamed Khaled, Algebras of Concepts and Their Networks: Boolean Algebras

47. Peter Koepke, The Naproche natural language proof assistant

48. Katarzyna W. Kowalik, Classifying Ramsey-theoretic principles with strongly infinite witnesses over RCA_0^*

49. Agnieszka Kozdęba and Apoloniusz Tyszka, The physical limits of computation inspire an open problem that concerns decidable sets $X \subseteq \mathbb{N}$ and cannot be formalized in ZFC as it refers to the current knowledge on X

50. Beibut Kulpeshov, On criterion for binarity of almost ω-categorical weakly o-minimal theories

51. Borisa Kuzeljevic and Stevo Todorcevic, Cofinal types on ω_2

52. Paul Blain Levy, Broad Infinity and Generation Principles

53. Ivan Di Liberti, Formal model theory and Higher Topology

54. Philipp Moritz Lücke, Huge reflection

55. Pawel Lupkowski, Erotetic search scenarios and blackboard architecture in group question decomposition

56. Mateusz Łełyk, On Sigma1 uniform reflection over uniform Tarski biconditionals
57. Judit Madarász, Concept algebra of special relativistic spacetime
58. Adam Malinowski and Ludomir Newelski, A few remarks on strongly generic sets
59. Nurlan Markhabatov and Sergey Sudoplatov, On closures for partially ordered families of theories
60. Santiago Jockwich Martinez, Algebra-valued models and Priests Logic of Paradox
61. Alba Massolo and Inés Crespo, Arguments against a Bayesian approach to the normativity of argumentation
62. Yukihiro Masuoka and Makoto Tatsuta, Counterexample to cut-elimination in cyclic proof system
63. Lachlan McPheat, Mehrnoosh Sadrzadeh, Hadi Wazni and Gijs Wijnholds, Vectorial discourse analysis in Lambek calculus with a bounded relevant modality
64. José M. Méndez, Gemma Robles and Francisco Salto Three-valued relevance logics
65. Yana Michailovskaya, Computable linear orders enriched by the relations S_L^n
66. Douglas Moore, Naturality as Universal Normative Authority in Stoic Logic
67. Sandra Müller, The strength of determinacy when all sets are universally Baire
68. Omer Ben Neria and Dominik Adolf, Tree-like scales and free subsets of set theoretic algebras
69. Yuya Okawa, Sohei Iwata and Taishi Kurahashi, Craig's interpolation and the fixed point properties for sublogics of interpretability logic IL.
70. Mattias Granberg Olsson and Graham Leigh, A proof of conservativity of $\hat{ID}_1^i$ over Heyting arithmetic via truth
71. Sergei Ospichev, About families in Ershov hierarchy without Friedberg numberings

72. Valeria de Paiva and Samuel G. Da Silva, Dialectica and Kolmogorov-Veloso problems

73. Inessa Pavlyuk and Sergey Sudoplatov, On rich properties for the family of theories of Abelian groups

74. Luiz Carlos Pereira, Elaine Pimentel and Valeria de Paiva, Ecumenic Negation: one or two?

75. Iosif Petrakis, Chu representations of categories related to constructive mathematics

76. Yaroslav Petrukhin, Cut-free proof systems for non-standard modal logics based on S5

77. Ivo Pezlar, A Note on Paradoxical Propositions from an Inferential Point of View

78. Paweł Płaczek The PTIME complexity of multiplicative nonassociative bilinear logic

79. Alexej Pynko, Finite Hilbert-style calculi for disjunctive and implicative finitely-valued logics with equality determinant

80. Davide Emilio Quadrellaro and Gianluca Grilletti, Topological Semantics for Inquisitive and DNA-logics

81. Paula Quinon, The anti-mechanist argument based on Gödel's Incompleteness Theorems, indescribability of the concept of natural number and deviant encodings

82. Eric Raidl, Definable conditionals

83. Eric Raidl, Andrea Iacona and Vincenzo Crupi, The Logic of the Evidential Conditional

84. Luca Reggio, Game comonads and homomorphism counting in finite model theory

85. Gemma Robles, Alternative semantical interpretations of the paraconsistent and paracomplete 4-valued logic PŁ4

86. Lorenzo Rossi and Michał Tomasz Godziszewski, First-order vs. Second-order theories: searching for deep disagreement

87. Yana Rumenova and Tinko Tinchev, Undecidability of modal definability: the class of frames with two commuting equivalence relations

88. Francisco Salto and Carmen Requena, BRAIN ACTIVITY MARKS OF LOGICAL VALIDITY: results from EEG and MEG studies

89. Sam Sanders, The Big Six and Big Seven of Reverse Mathematics, and a new zoo

90. Anton Setzer, A model of computation for single threaded sequential interactive programs

91. Paul Shafer, An infinite Π_1 set with no Δ_2 cohesive subset

92. Michał Sochański, Dorota Leszczyńska-Jasion, Szymon Chlebowski, Agata Tomczyk, Aleksander Kiryk and Marcin Jukiewicz, Synthetic tableaux: using data analysis in the optimization of proof search

93. Michał Sochański and Dorota Leszczyńska-Jasion, On the representation of logical formulas as cographs

94. William Stafford, Completeness of intuitionistic logic for generalised proof-theoretic semantics

95. Raffael Stenzel, $(\infty, 1)$-Categorical Comprehension Schemes

96. Alexey Stukachev, Marina Stukacheva and Alexey Ryzhkov, Approximation spaces over dense linear orders

97. Donald Stull, Algorithmic Randomness and Fractal Geometry

98. Gergely Székely, Conceptual Distance and Algebras of Concepts

99. Amirhossein Akbar Tabatabai, Feasible Visser-Harrop property for intuitionistic modal logics

100. Sourav Tarafder and Giorgio Venturi, ZF between classicality and non-classicality

101. Alberto Termine, Fabio Aurelio D'Asaro and Giuseppe Primiero, Modelling depth-bounded Boolean reasoning with Markov decision processes and reinforcement learning

102. Agata Tomczyk, Marta Gawek and Szymon Chlebowski, Natural Deduction Systems for Intuitionistic Logic with Identity

103. Christopher Turner, Forcing axioms and name principles

104. Sara L. Uckelman, Women in the history of logic: why does it matter who our foremothers are?

105. Michal Walicki Logic of sentential predicates

106. Bartosz Wcisło, Disjunctive correctness and sequential induction

107. Trevor Wilson, Characterizing strong cardinals, virtually strong cardinals, and other large cardinals by Löwenheim–Skolem properties

108. Urszula Wybraniec-Skardowska, A formal-logic approach to the ontology of language

109. Manlio Valenti, Algebraic properties of the first-order part of a problem

110. Giorgio Venturi, On non-classical models of ZFC

111. Viktor Verbovskiy and Aigerim Dauletiyarova, On local monotonicity of unary functions definable in o-stable ordered groups

112. Jannik Vierling, The limits of the n-clause calculus

113. Aibat Yeshkeyev, Aigul Issayeva and Nazgul Shamatayeva, On atomic and algebraically prime definable subsets of semantic model

114. Aibat Yeshkeyev, Olga Ulbrikht and Nazerke Mussina, On the categoricity of the class of the Jonsson spectrum

115. Aibat Yeshkeyev and Olga Ulbrikht, The Jonsson nonforking notion under some positiveness

116. Pedro H. Zambrano and David Reyes, Co-quantale valued logics

117. Maxim Zubkov and Andrey Frolov, Spectral universality of linear orders with one binary relation.

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1. Pavel Arazim, Logic not being serious
2. Bruno Bentzen, How do we intuit mathematical constructions?
3. Nick Bezhanishvili and Fan Yang, Intermediate logics in the team semantics setting
4. Ludovica Conti, Logicality and Abstraction
5. Azza Gaysin, H-Coloring Dichotomy in Proof Complexity
6. Michał Tomasz Godziszewski and Luca San Mauro, Quotient structures, philosophy of computability theory and computational structuralism
7. Michał Tomasz Godziszewski, Fairness and Jutsified Representation in Judgment Aggregation and Belief Merging
8. Mohamed Khaled, Algebras of Concepts and Their Networks: Boolean Algebras
9. Judit Madarász, Concept algebra of special relativistic spacetime
10. José M. Méndez, Gemma Robles and Francisco Salto Three-valued relevance logics
11. Iosif Petrakis, Chu representations of categories related to constructive mathematics
12. Paula Quinon, The anti-mechanist argument based on Gödel's Incompleteness Theorems, indescribability of the concept of natural number and deviant encodings
13. Lorenzo Rossi and Michał Tomasz Godziszewski, First-order vs. Second-order theories: searching for deep disagreement
14. William Stafford, Completeness of intuitionistic logic for generalised proof-theoretic semantics
15. Raffael Stenzel, $(\infty, 1)$-Categorical Comprehension Schemes
16. Alberto Termine, Fabio Aurelio D'Asaro and Giuseppe Primiero, Modelling depth-bounded Boolean reasoning with Markov decision processes and reinforcement learning

17. Sara L. Uckelman, Women in the history of logic: why does it matter who our foremothers are?

18. Jannik Vierling, The limits of the n-clause calculus

19. Michal Walicki, Logic of sentential predicates

Computability

1. Nikolay Bazhenov, Dariusz Kalociński and Michał Wrocławski, Degree spectra of unary recursive functions on naturals with standard ordering

2. Laurent Bienvenu, Valentino Delle Rose and Tomasz Steifer, Computable randomness relative to almost all oracles

3. Marta Fiori Carones, Measuring the strength of Ramsey-theoretic statements over $RCA0^*$

4. Vittorio Cipriani, Cantor-Bendixson theorem in the Weihrauch lattice

5. Andrés Cordón-Franco, F. Félix Lara-Martín and Manuel J.S. Loureiro, On determinacy of Lipschitz and Wadge games in second order arithmetic

6. Karol Duda and Aleksander Iwanow, A finitely presented group with undecidable amenability

7. Margarita Gaskova, Boolean algebras autostable relative to n-constructivizations

8. Josiah Jacobsen-Grocott, A Characterization of the Strongly η-Representable Many-One Degrees

9. Katarzyna W. Kowalik, Classifying Ramsey-theoretic principles with strongly infinite witnesses over RCA_0^*

10. Agnieszka Kozdęba and Apoloniusz Tyszka, The physical limits of computation inspire an open problem that concerns decidable sets $X \subseteq \mathbb{N}$ and cannot be formalized in ZFC as it refers to the current knowledge on X

11. Yana Michailovskaya, Computable linear orders enriched by the relations S_L^n

12. Sergei Ospichev, About families in Ershov hierarchy without Friedberg numberings

13. Paweł Płaczek The PTIME complexity of multiplicative nonassociative bilinear logic

14. Sam Sanders, The Big Six and Big Seven of Reverse Mathematics, and a new zoo
15. Anton Setzer, A model of computation for single threaded sequential interactive programs
16. Paul Shafer, An infinite Π_1 set with no Δ_2 cohesive subset
17. Donald Stull, Algorithmic Randomness and Fractal Geometry
18. Manlio Valenti, Algebraic properties of the first-order part of a problem
19. Maxim Zubkov and Andrey Frolov, Spectral universality of linear orders with one binary relation.

Logic in Cognitive Science and Linguistics

1. Jakub Dakowski, Aleksandra Draszewska, Barbara Adamska, Dominika Juszczak, Łukasz Abramowicz and Robert Szymański, Addressing logic students' proof making difficulties with Plugin Oriented Programming and gamification
2. Andrzej Gajda, Abductive reasoning in a neural-symbolic system
3. Pawel Lupkowski, Erotetic search scenarios and blackboard architecture in group question decomposition
4. Alba Massolo and Inés Crespo, Arguments against a Bayesian approach to the normativity of argumentation
5. Lachlan McPheat, Mehrnoosh Sadrzadeh, Hadi Wazni and Gijs Wijnholds, Vectorial discourse analysis in Lambek calculus with a bounded relevant modality
6. Francisco Salto and Carmen Requena, BRAIN ACTIVITY MARKS OF LOGICAL VALIDITY: results from EEG and MEG studies
7. Urszula Wybraniec-Skardowska, A formal-logic approach to the ontology of language

Modal and Epistemic Logic

1. Aleksi Anttila, Maria Aloni and Fan Yang, A logic for modelling free choice inference
2. Gaia Belardinelli and Rasmus K. Rendsvig, Epistemic Planning with Attention as a Bounded Resource

3. Dylan Bellier, Massimo Benerecetti, Dario Della Monica and Fabio Mogavero, Good-for-Game QPTL: An Alternating Hodges Semantics
4. Willem Conradie and Valentin Goranko, Algorithmic correspondence for relevance logics
5. Meghdad Ghari, A temporal logic of justification and obligation
6. Taneli Huuskonen, Cromulence logic: duty meets preference
7. Douglas Moore, Naturality as Universal Normative Authority in Stoic Logic
8. Yuya Okawa, Sohei Iwata and Taishi Kurahashi, Craig's interpolation and the fixed point properties for sublogics of interpretability logic IL.
9. Davide Emilio Quadrellaro and Gianluca Grilletti, Topological Semantics for Inquisitive and DNA-logics
10. Eric Raidl, Definable conditionals
11. Eric Raidl, Andrea Iacona and Vincenzo Crupi, The Logic of the Evidential Conditional
12. Gemma Robles, Alternative semantical interpretations of the paraconsistent and paracomplete 4-valued logic PŁ4
13. Yana Rumenova and Tinko Tinchev, Undecidability of modal definability: the class of frames with two commuting equivalence relations

Model Theory

1. Claudio Agostini and Eugenio Colla, An algebraic characterization of Ramsey Monoids
2. Aizhan Altayeva, Beibut Kulpeshov and Sergey Sudoplatov, On algebras of binary formulas for almost ω-categorical weakly o-minimal theories
3. John Baldwin, Finer classification of Strongly minimal sets
4. Dmitry Emelyanov, Beibut Kulpeshov and Sergey Sudoplatov, On algebras of binary formulas for partially ordered theories
5. Christian Espindola, Categoricity theorems in infinite quantifier languages
6. Francesco Gallinaro, Around exponential algebraic closedness
7. Davit Harutyunyan, On Some Associative Formula with Functional Variables

8. Ruaan Kellerman and Valentin Goranko, Approximating trees as coloured linear orders and complete axiomatisations of some classes of trees
9. Beibut Kulpeshov, On criterion for binarity of almost ω-categorical weakly o-minimal theories
10. Ivan Di Liberti, Formal model theory and Higher Topology
11. Adam Malinowski and Ludomir Newelski, A few remarks on strongly generic sets
12. Nurlan Markhabatov and Sergey Sudoplatov, On closures for partially ordered families of theories
13. Inessa Pavlyuk and Sergey Sudoplatov, On rich properties for the family of theories of Abelian groups
14. Luca Reggio, Game comonads and homomorphism counting in finite model theory
15. Alexey Stukachev, Marina Stukacheva and Alexey Ryzhkov, Approximation spaces over dense linear orders
16. Gergely Székely, Conceptual Distance and Algebras of Concepts
17. Viktor Verbovskiy and Aigerim Dauletiyarova, On local monotonicity of unary functions definable in o-stable ordered groups
18. Aibat Yeshkeyev, Aigul Issayeva and Nazgul Shamatayeva, On atomic and algebraically prime definable subsets of semantic model
19. Aibat Yeshkeyev, Olga Ulbrikht and Nazerke Mussina, On the categoricity of the class of the Jonsson spectrum
20. Aibat Yeshkeyev and Olga Ulbrikht, The Jonsson nonforking notion under some positiveness
21. Pedro H. Zambrano and David Reyes, Co-quantale valued logics

Proofs and Programs

1. Katalin Bimbo, Abaci running backward
2. Nicola Bonatti, Two questions concerning quantifiers rules
3. Yong Cheng, The interpretation degree structure of r.e. theories for which the first incompleteness theorem holds

4. Horatiu Cheval, General metatheorems in proof mining
5. Anahit Chubaryan and Arsen Hambardzumyan, On non-monotonous properties of some propositional proof systems
6. Anahit Chubaryan, Sargis Hovhannisyan and Hayk Gasparyan, Comparison of two propositional proof systems by lines and by sizes
7. Lew Gordeev and Edward Haeusler, On Proof Theory in Computational Complexity
8. Raheleh Jalali, On hard theorems for substructural logics
9. Marcin Jukiewicz and Dorota Leszczyńska-Jasion, Improving the work of a genetic algorithm in proof-search tasks
10. Peter Koepke, The Naproche natural language proof assistant
11. Mateusz Łełyk, On Sigma1 uniform reflection over uniform Tarski biconditionals
12. Yukihiro Masuoka and Makoto Tatsuta, Counterexample to cut-elimination in cyclic proof system
13. Mattias Granberg Olsson and Graham Leigh, A proof of conservativity of $\hat{ID}^i_1$ over Heyting arithmetic via truth
14. Valeria de Paiva and Samuel G. Da Silva, Dialectica and Kolmogorov-Veloso problems
15. Luiz Carlos Pereira, Elaine Pimentel and Valeria de Paiva, Ecumenic Negation: one or two?
16. Yaroslav Petrukhin, Cut-free proof systems for non-standard modal logics based on S5
17. Ivo Pezlar, A Note on Paradoxical Propositions from an Inferential Point of View
18. Alexej Pynko, Finite Hilbert-style calculi for disjunctive and implicative finitely-valued logics with equality determinant
19. Michał Sochański and Dorota Leszczyńska-Jasion, On the representation of logical formulas as cographs
20. Michał Sochański, Dorota Leszczyńska-Jasion, Szymon Chlebowski, Agata Tomczyk, Aleksander Kiryk and Marcin Jukiewicz, Synthetic tableaux: using data analysis in the optimization of proof search

21. Amirhossein Akbar Tabatabai, Feasible Visser-Harrop property for intuitionistic modal logics

22. Agata Tomczyk, Marta Gawek and Szymon Chlebowski, Natural Deduction Systems for Intuitionistic Logic with Identity

23. Bartosz Wcisło, Disjunctive correctness and sequential induction

Set Theory

1. Matteo de Ceglie, The V-logic Multiverse and MAXIMIZE

2. Gabriel Ciobanu, Various notions of infinity for finitely supported structures

3. Luke Gardiner, Countable Exponent Partition Relations on the Real Line

4. Martina Iannella, The complexity of convex bi-embeddability among countable linear orders

5. Vladimir Kanovei, On the 'Definability of definable' problem of Alfred Tarski

6. Borisa Kuzeljevic and Stevo Todorcevic, Cofinal types on ω_2

7. Paul Blain Levy, Broad Infinity and Generation Principles

8. Philipp Moritz Lücke, Huge reflection

9. Santiago Jockwich Martinez, Algebra-valued models and Priests Logic of Paradox

10. Sourav Tarafder and Giorgio Venturi, ZF between classicality and non-classicality

11. Christopher Turner, Forcing axioms and name principles

Keynotes

- Linda Westrick, Pennsylvania State University
- Benoit Monin, Créteil University
- Noam Greenberg, Victoria University of Wellington
- Vera Fischer, University of Viena
- Luca Motto Ros, University of Turin
- Elaine Pimentel, Federal University of Rio Grande do Norte
- Frank Pfenning, Carnegie Mellon University
- Johan van Benthem, University of Amsterdam
- Ryan Williams, Massachusetts Institute of Technology
- Artem Chernikov, University of California Los Angeles

▸ JOHAN VAN BENTHEM (JOINT WORK WITH THOMAS ICARD), *Interleaving Logic and Counting, A View from the Bottom.*
University of Amsterdam.
E-mail: `johan@stanford.edu`.

Reasoning with quantifier expressions in natural language combines logical and arithmetical features, transcending divides between qualitative and quantitative. Our topic is this cooperation of styles. A view from the top is afforded by FO(#), first-order logic with counting operators and cardinality comparisons. This system is known to be of very high complexity, and drowns out finer aspects of the combination of logic and counting. Therefore, we start from the bottom, and move to a small fragment that can represent numerical syllogisms and basic reasoning about comparative size: monadic first-order logic with counting (MFO(#)). We provide normal forms that allow for axiomatization, determine which arithmetical notions can be defined on finite and (using a separation theorem) on infinite models, and conversely, we discuss which logical notions can be defined out of purely arithmetical ones, and what sort of (non-)classical logics can be induced.

Next we investigate a series of strengthenings of MFO(#) using the same methods. The second-order version is close to additive Presburger Arithmetic, while versions with tuple counting take us to Diophantine equations, making the logic undecidable. We also define a system ML(#) that combines basic modal logic over binary accessibility relations with counting, needed to formulate ubiquitous reasoning patterns such as the Pigeon Hole Principle. We prove decidability of ML(#), and provide a new kind of bisimulation matching the expressive power of the language.

As a complement to the fragment approach pursued here, we also discuss another way of lowering complexity: changing the semantics of counting in natural ways, such as allowing dependencies between variables, or introducing well-motivated abstract values for 'mass' or other aggregating notions than cardinalities.

Finally, we return to natural language, confronting the architecture of our formal systems with linguistic quantifier vocabulary and reasoning modules such as the monotonicity calculus. In addition, we discuss the role of counting in evaluation procedures for quantifier expressions and determine which binary quantifiers are computable by finite 'semantic automata'. We conclude with some general thoughts on further entanglements of logic and counting in formal systems, on rethinking the qualitative/quantitative divide, and on empirical consequences of our analysis.

Caveat. Logic and counting is a topic with a vast scattered literature. The full paper will contain many references, especially to relevant papers in computational logic.

▸ ARTEM CHERNIKOV, *Measures in model theory.*

University of California Los Angeles.
E-mail: chernikov@math.ucla.edu.

In model theory, a type is an ultrafilter on the Boolean algebra of definable sets in a structure, which is the same thing as a finitely additive $\{0,1\}$-valued measure. This is a special kind of a Keisler measure, which is just a finitely additive real-valued probability measure on the Boolean algebra of definable sets. Introduced by Keisler in the late 80's, Keisler measures became a central object of study in the last decade. This is motivated by several intertwined lines of research. One of them (and perhaps the oldest one) is the development of probabilistic and continuous logics. Another is the study of definable groups in o-minimal, and more generally in NIP theories, leading to interesting connections with topological dynamics. Further motivation comes from applications in additive and in extremal combinatorics, uniting the aforementioned directions. I will survey some of the recent developments in the subject.

▶ VERA FISCHER, *Combinatorial sets of reals.*
University of Viena.
E-mail: vera.fischer@univie.ac.at.

Infinitary combinatorial sets of reals, such as almost disjoint families, cofinitary groups, independent families, and towers occupy a central place in the study of the set-theoretic properties of the real line. Of particular interest are such extremal sets of reals, i.e. combinatorial sets which are maximal under inclusion with respect to a desired property, their possible cardinalities, definability properties, as well as the existence or non-existence of ZFC dependences. The study of such combinatorial sets of reals is closely connected with the development of a broad spectrum of forcing techniques.

In this talk we will see some recent advances in the subject and point towards interesting remaining open questions.

▸ NOAM GREENBERG, *The information common to relatively random sequences.*
Victoria University of Wellington.
E-mail: noam.greenberg@vuw.ac.nz.

If X and Y are relatively random, what common information can X and Y have? We use algorithmic randomness and computability theory to make sense of this question. The answer involves some unexpected ingredients, such as the Lebesgue density theorem, and linear programming, and reveals a rich hierarchy of Turing degrees within the K-trivial degrees.

▶ BENOIT MONIN, *The computational content of Milliken's tree theorem.*
Créteil University.
E-mail: benoit.monin@computability.fr.
The Milliken's tree theorem is an extension of Ramsey's theorem to trees. It implies for instance that if we assign to all the sets of two strings of the same length, one mong k colors, there is an infinite binary tree within which every pair of strings of the same height has the same color. We are going to present some results on Milliken's tree theorem from the viewpoint of computability theory and reverse mathematics.

▶ FRANK PFENNING, *Adjoint Logic*.
Carnegie Mellon University.
E-mail: `fp@cs.cmu.edu`.

We introduce adjoint logic as a general framework for integrating logics with different structural properties, that is, admitting or denying exchange, weakening, or contraction among the hypotheses. We investigate its proof-theoretic properties from two angles: proof construction and proof reduction. The former is the basis for applications in logical frameworks and logic programming, while the latter provides computational interpretations in functional and concurrent programming. We briefly sketch some of these applications.

The talk presents joint work with William Chargin, Klaas Pruiksma, and Jason Reed.

► ELAINE PIMENTEL, *A pure view of ecumenical modalities.*
Federal University of Rio Grande do Norte.
E-mail: elaine.pimentel@gmail.com.

The discussion about how to put together Gentzen's systems for classical and intuitionistic logic in a single unified system is back in fashion. Indeed, recently Prawitz and others have been discussing the so-called ecumenical Systems, where connectives from these logics can co-exist in peace. In Prawitz' system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation, and the constant for the absurd, but they would each have their own existential quantifier, disjunction, and implication, with different meanings.

In this talk, we show how to extend Prawitz' ecumenical idea to alethic K-modalities: using Simpson's meta-logical characterization, necessity is independent of the viewer, while possibility can be either intuitionistic or classical. We will show an internal pure calculus for ecumenical modalities, nEK, where every basic object of the calculus can be read as a formula in the language of the ecumenical modal logic EK. We prove that nEK is sound and complete w.r.t. the ecumenical birrelational semantics, and discuss fragments and modal extensions.

This is a joint work with Sonia Marin, Luiz Carlos Pereira and Emerson Sales.

▸ LUCA MOTTO ROS, *Generalized descriptive set theory for all cofinalities, and some applications.*

University of Turin.
E-mail: `luca.mottoros@unito.it`.

Generalized descriptive set theory is nowadays a very active field of research. The idea is to develop a higher analogue of classical descriptive set theory in which ω is systematically replaced with an uncountable cardinal κ. With a few exceptions, papers in this area tend to concentrate on the case of regular cardinals. This is because under such assumption one can easily generalize a number of basic facts and techniques from the classical setup, but from the theoretical viewpoint the choice is indeed not fully justified.

In this talk I will survey some recent work in which the theory is instead developed in a uniform and cofinality-independent way, thus naturally including the case of singular cardinals. I will also consider some interesting applications connecting generalized descriptive set theory to Shelah's stability theory (in the case of regular cardinals), and to the study of nonseparable complete metric spaces under Woodin's axiom $\mathsf{I0}$ (in the case of singular cardinals of countable cofinality).

▸ LINDA WESTRICK, *Reverse mathematics of Borel sets.*
Pennsylvania State University.
E-mail: westrick@psu.edu.

Theorems about Borel sets are often proved using arguments which appeal to some regularity property of Borel sets, rather than recursing on the Borel structure of the set directly. For example, the statement "there is no Borel well-ordering of the reals" can be proved using either a measure or category argument. More generally, suppose we are given a theorem about Borel sets and a proof based on their measurability. Could the same theorem also be proved with a category argument? In principle, when the answer is "no", reverse mathematics provides a framework for proving this negative answer. However, if ATR is taken as a base theory, measure and category arguments cannot be distinguished. That is because both "Every Borel set is measurable" and "Every Borel set has the property of Baire" follow already from ATR.

The notion of a completely determined Borel set, which is now a few years old, allows theorems involving Borel sets to be analyzed over a weaker base theory. The principles "Every c.d. Borel set is measurable" and "Every c.d. Borel set has the property of Baire" are both strictly weaker than ATR and incomparable with each other. With reference to these landmarks, we present what is known about the reverse mathematical strength of weak theorems involving Borel sets, including the Borel Dual Ramsey Theorem and some theorems from descriptive combinatorics. We also characterize the sets which HYP believes are c.d. Borel.

This work was partially supported by NSF grant DMS-1854107, and parts are joint with various collaborators: Astor, Dzhafarov, Flood, Montalban, Solomon, Towsner and Weisshaar.

▸ RYAN WILLIAMS, *Complexity Lower Bounds from Algorithm Design.*
Massachusetts Institute of Technology.
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Since the beginning of the theory of computation, researchers have been fascinated by the prospect of proving impossibility results on computing. When and how can we argue that a task cannot be efficiently solved, no matter what algorithm we use? I will briefly introduce some of the ideas behind a research program in computational complexity that I and others have studied, for the last decade. The program begins with the observations that:

(a) Computer scientists know a great deal about how to design efficient algorithms.

(b) However, we do not know how to prove many weak-looking complexity lower bounds.

It turns out that certain knowledge we have from (a) can be leveraged to prove complexity lower bounds in a systematic way, making progress on (b). For example, progress on faster circuit satisfiability algorithms (even those that barely improve upon exhaustive search) automatically imply circuit complexity lower bounds for interesting functions.

Gödel Lecture

Elisabeth Bouscaren - CNRS – Université Paris-Sud

▸ ELISABETH BOUSCAREN, *The ubiquity of configurations in Model Theory.*
CNRS – Université Paris-Sud.
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Originally in Classification Theory, then in Geometric Stability and now, beyond Stability, in Tame Model Theory, one common essential feature is the identification and study of some geometric configurations, of combinatorial and dimensional theoretic nature. They can witness the combinatorial and the model theoretic complexity of a theory or indicate the existence of specific definable algebraic structures. This enables model theory to tackle questions from very diverse subjects.

We will attempt to illustrate the importance of these configurations through some examples

Tutorials

- Krzysztof Krupiński, University of Wrocław
- Andrew Marks, University of California Los Angeles

▶ KRZYSZTOF KRUPIŃSKI, *Topological dynamics in model theory.*
University of Wrocław.
E-mail: kkrup@math.uni.wroc.pl.

Some fundamental notions and methods of topological dynamics were introduced to model theory by Newelski in the mid 2000's.

In the first part of my tutorial, I will recall some basic notions of topological dynamics, discuss the flows which appear naturally in model theory (as various spaces of types), and give applications of basic topological dynamics to some group covering results of Newelski such as: if an $\aleph_0$-saturated group is covered by countably many 0-type-definable sets, X_n, $n \in \omega$, then for some finite $A \subseteq G$ and $n \in \omega$, $G = AX_nX_n^{-1}$.

In the second part, I will define the Ellis semigroup and Ellis group of a flow, and focus on connections between the Ellis groups of natural flows in model theory and certain invariants of definable groups (quotients by model-theoretic connected components) or first order theories (Galois groups of first order theories as well as spaces of strong types). In particular, I will discuss the results of Pillay, Rzepecki and myself which present certain invariants of this kind as quotients of compact (Hausdorff) groups (which are canonical Hausdorff quotients of Ellis groups). This has various consequences obtained by Pillay, Rzepecki and myself, e.g. it leads to a general result that model-theoretic *type-definability* of a bounded invariant equivalence relation defined on a single complete type over $\emptyset$ is equivalent to descriptive set theoretic *smoothness* of this relation.

In the last part, I will discuss a definable variant of Kechris, Pestov, Todorčević (KPT) theory, developed by Lee, Moconja and myself. KPT theory studies relationships between dynamical properties of the groups of automorphisms of Fraïssé structures and Ramsey-theoretic (so combinatorial) properties of the underlying Fraïssé classes. In our research, the idea is to find interactions between dynamical properties of first order theories (i.e. properties related to the actions of the automorphism group of a sufficiently saturated model on various types spaces over this model) and definable versions of Ramsey-theoretic properties of the theory. This leads to analogs of various results of KPT theory (i.e. a combinatorial characterization of the definable extreme amenability of a theory), but also to some rather novel theorems, e.g. yielding criteria for profiniteness of the Ellis group of a first order theory.

▸ ANDREW MARKS, *Characterizing Borel complexity and an application to decomposability.*
University of California Los Angeles.
E-mail: marks@math.ucla.edu.
We give a new characterization of when sets in the Borel hierarchy are Σ_n^0 hard. This characterization is proved using Antonio Montalban's true stages method for conducting priority arguments in computability theory. We use this to prove the decomposability conjecture, assuming projective determinacy. The decomposability conjecture describes what Borel functions are decomposable into a countable union of partial continuous functions with $\Pi_n^0 domains$. This is joint work with Adam Day.

Sessions

CMP Computability

Invited talks

ORGANIZERS:
Steffen Lempp
Matthew Harrison-Trainor

INVITED SPEAKERS:
Nikolai Bazhenov, Sobolev Institute of Mathematics (bazhenov@math.nsc.ru)
Leszek Kołodziejczyk, University of Warsaw (lak@mimuw.edu.pl)
Jun Le Goh, Cornell University (junle.goh@wisc.edu)
Arman Darbinyan, Texas A&M University (adarbina@math.tamu.edu)

▸ NIKOLAY BAZHENOV, *Primitive recursive algebraic structures, and the theory of numberings.*

E-mail: bazhenov@math.nsc.ru.

The paper of Kalimullin, Melnikov, and Ng (2017) was a starting point for the recent significant progress in the studies of sub-recursive algebra. The key notion in these developments is that of a punctual structure. A countably infinite structure S is punctual if the domain of S is the set of natural numbers, and the signature predicates and functions of S are uniformly primitive recursive. In the talk, we discuss recent results on punctual algebraic structures, and related results on upper semilattices of numberings.

▶ LESZEK KOLODZIEJCZYK, *Weak König's Lemma in the absence of Σ^0_1 induction.*
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Reverse mathematics studies the logical strength of mathematical theorems by proving implications between the theorems and some well-established set existence principles expressed in the language of second-order arithmetic. The implications are proved over a fixed base theory embodying "computable mathematics". The usual base theory, RCA_0, is axiomatized by comprehension for computable (i.e. Δ^0_1-definable) properties of natural numbers and by induction for c.e. (i.e. Σ^0_1-definable) properties. In the 1980's, Simpson and Smith introduced an alternative weaker base theory RCA^*_0, in which Σ^0_1-induction is replaced by Δ^0_1-induction.

One of the most important set existence principles considered in reverse mathematics is Weak König's Lemma, WKL. This states that every infinite binary tree has an infinite branch. Since there are computable binary trees without computable branches, WKL is not provable in RCA_0, but it is well-known that adding WKL to RCA_0 results in a theory that does not prove any new Π^1_1 statements.

Already Simpson and Smith showed that an analogous Π^1_1-conservation result for WKL also holds over RCA^*_0. We prove that WKL nevertheless behaves very differently over $\mathrm{RCA}^*_0 + \neg\mathrm{RCA}_0$ than in the traditional setting. Namely, any two countable models of $\mathrm{RCA}^*_0 + \mathrm{WKL}$ that have a common first-order part and share a common witness to the failure of Σ^0_1-induction are isomorphic. It follows, for instance, that WKL is the strongest Π^1_2 statement that is Π^1_1-conservative over $\mathrm{RCA}^*_0 + \neg\mathrm{RCA}_0$. Moreover, the isomorphism theorem provides new information about the structure of models of RCA_0 that satisfy Δ^0_2- but not Σ^0_2-induction, which has some implications for traditional reverse mathematics.

Joint work with Marta Fiori Carones, Tin Lok Wong, and Keita Yokoyama.

► JUN LE GOH, *Redundancy of information: Lowering effective dimension.*
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The effective Hausdorff dimension of an infinite binary sequence can be characterized using the (normalized) Kolmogorov complexity of its initial segments (Mayordomo). It is invariant under changes on a set of positions of upper density 0. Greenberg, Miller, Shen, and Westrick initiated the study of how effective Hausdorff dimension can be changed if one is allowed to change a sequence on a set of positive upper density. Specifically, given some X of dimension t, what is the minimum density of changes needed to obtain some Y of dimension s? The situation differs depending on X, as well as the value of the target dimension s relative to the value of the starting dimension t. We present joint work with Miller, Soskova and Westrick on these questions.

► ARMAN DARBINYAN, *Computable groups and computable group orderings.*
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An important class of abstract groups is the one that consists of linearly ordered groups whose orders are invariant under left (and right) group multiplications. From computability point of view it is interesting to investigate when orderable groups admit computable orders. In particular, a question of Downey and Kurtz asks about existence of computable orderable groups that do not admit computable orders with respect to any group presentation. In my talk I will discuss recent advancements on this topic.

Contributed talks

▸ NIKOLAY BAZHENOV, DARIUSZ KALOCIŃSKI, MICHAŁ WROCŁAWSKI, *Degree spectra of unary recursive functions on naturals with standard ordering.*
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Existing results provide some insights into obtaining computable, c.e. or Δ_2 degrees as a spectrum of a *unary* recursive function on naturals with standard ordering $\leq$ [1, 3]. We extend these results by providing a more complete picture, covering natural subclasses of unary recursive functions. For example, we show that if a computable structure $(\omega, \leq, f)$ has a finitely generated infinite substructure, then the degree spectrum of f on $(\omega, \leq)$ contains precisely c.e. degrees. This prompts to introduce the notion of an f-block, understood as a substructure of $(\omega, \leq, f)$, with the domain equal to some interval in $(\omega, \leq)$ and with no proper substructures. We will discuss the following result: if a computable structure $(\omega, \leq, f)$, with f unary, has only finitely many isomorphism types of f-blocks, and all its f-blocks are finite, then either f is intrinsically computable or its degree spectrum on $(\omega, \leq)$ consists of all Δ_2 degrees.

We also briefly discuss the philosophical side of the results which becomes more apparent when viewed through the lens of Shapiro's notations for natural numbers [2]. For example, Shapiro's notion of a function computable in every notation (with computable ordering) coincides with functions having the trivial degree spectrum. Our results provide a more fine grained classification of the complexity of functions in various notations, as measured by the Turing degrees thereof.

[1] Rod Downey, Bakhadyr Khoussainov, Joseph S. Miller, and Liang Yu, *Degree spectra of unary relations on* $(\omega, <)$, ***Logic, Methodology and Philosophy of Science*** (Clark Glymour, Wang Wei, and Dag Westerstahl, editors), College Publications, London, 2009, pp. 35–55.

[2] Stewart Shapiro, *Acceptable notation*, ***Notre Dame Journal of Formal Logic***, vol. 23 (1982).

[3] Matthew Wright, *Degrees of relations on ordinals*, ***Computability***, vol. 7 (2018).

▸ LAURENT BIENVENU, VALENTINO DELLE ROSE, AND TOMASZ STEIFER,

Computable randomness relative to almost all oracles.

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It follows from van Lambalgen's theorem for Martin-Löf randomness, that every Martin-Löf random set X is also Martin-Löf random relative to almost all oracles. Is this also true for notions of randomness for which van Lambalgen's theorem does not hold? We answer this question in the negative for computable randomness. A binary sequence X is a.e. computably random if there is no probabilistic computable strategy which is total and succeeds on X for positive measure of oracles. Using the fireworks technique we construct a sequence which is computably random but not a.e. computably random. We also prove separation between a.e. computable randomness and partial computable randomness. This happens exactly in the uniformly almost everywhere dominating Turing degrees.

[1] L. Yu, *When van Lambalgen's Theorem fails*, ***Proceedings of the American Mathematical Society***, 2007, 861-864.

[2] L. Bienvenu, L. Patey, *Diagonally non-computable functions and fireworks*, ***Information and Computation***, 2017, 253: 64-77.

▸ MARTA FIORI CARONES, *Measuring the strength of Ramsey-theoretic statements over* RCA_0^*.

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(Joint work with Leszek Kołodziejczyk and Katarzyna W. Kowalik)

The common base theory of reverse mathematics is the theory RCA_0, which guarantees the existence of Δ^0_1-definable sets and where mathematical induction for Σ^0_1-formulae holds. In 1986, Simpson and Smith introduced a different base theory, RCA_0^*, where induction is weakened to Δ^0_1-formulae. In more recent years Kołodziejczyk, Kowalik, Wong, Yokoyama started wondering about the strength of Ramsey's theorem over RCA_0^*. In this talk we concentrate on three well known consequences of Ramsey's theorem for pairs, namely the Ascending Descending Sequence principle ADS, the Chain/Antichain principle CAC and the Cohesive Ramsey theorem for pairs CRT^2_2. We measure the relative strength of these statements in three ways: (1) implications or non-implications among them over RCA_0^* (and over RCA_0^* plus negated Σ^0_1-induction), (2) conservativity over RCA_0^* and (3) provable closure properties of the intersection of all Σ^0_1-cuts. For example, we show that with respect to the last criterion, RT^2_2 is stronger than both CAC and ADS, while these two are indistinguishable, and it is still open whether CRT^2_2 resembles RT^2_2 or $\mathsf{CAC}/\mathsf{ADS}$.

▸ VITTORIO CIPRIANI, *Cantor-Bendixson theorem in the Weihrauch lattice.*
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In this talk, we continue the program initiated in [4] aiming to study theorems occurring at the high levels of reverse mathematics (see [1]). Recently there has been a growing interest in this topic by many authors, see for example [5], [2], [6], [7] and a recent survey with some open problems concerning also this specific area [8]. We first present some results at the level of ATR_0, showing that (a variant of) the perfect tree theorem (in [4] denoted with PTT_1) is strictly stronger than its version for closed sets, showing a difference with respect to the reverse mathematics setting where the two principles are equivalent. On the other hand, if one considers arithmetical Weihrauch reducibility (see [2] and [7]) the two principles are equivalent. We then move our attention to natural counterparts of the highest subsystem in the big five in reverse mathematics, namely Π^1_1-CA_0, focusing on principles related to the Cantor-Bendixson theorem. The natural function representing Π^1_1-CA_0 is the one that maps a countable sequence of trees to the characteristic function of the set of indices corresponding to well-founded trees. Recently in [3], Hirst showed its Weihrauch equivalence with PK, the function that takes as input a tree and outputs its perfect kernel. We will show that PK, as defined in [3], is strictly stronger than the version for closed sets, even if, as for the perfect tree theorem, they are arithmetically equivalent. Our analysis then moves to multivalued functions representing variations of the Cantor-Bendixson theorem, that, given in input a closed set output its perfect kernel plus a listing of the isolated points. We will show that (variations of) the Cantor-Bendixson theorem for trees are as strong as the perfect kernel theorem (for trees). The same holds for closed sets with the only exception regarding a version of the Cantor-Bendixson in Baire space.
This is joint work with Alberto Marcone and Manlio Valenti.

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[2] Paul-Elliot Angls d'Auriac, Takayuki Kihara, *A comparison of various analytic choice principles*, Available at https://arxiv.org/abs/1907.02769, 2019.

[3] Jeffry L. Hirst, *Leaf management*, ***Computability***, vol. 9 (2020), no. 3-4, pp.309–314.

[4] Takayuki Kihara, Alberto Marcone, Arno Pauly, *Searching for an analogue of* ATR_0 *in the Weihrauch lattice*, ***The Journal of Symbolic Logic***, vol. 85(3) (2020), no. 1006-1043, pp.1–37.

[5] Alberto Marcone, Manlio Valenti, *The open and clopen Ramsey theorems in the Weihrauch lattice*, ***The Journal of Symbolic Logic***, to appear,doi=10.1017/jsl.2021.105. Available at https://arxiv.org/abs/2003.04245

[6] Jun Le Goh, Arno Pauly, Manlio Valenti, *Finding descending sequences through ill-founded linear orders*, ***The Journal of Symbolic Logic***, to appear,doi=10.1017/jsl.2021.15. Available at https://arxiv.org/abs/2010.03840

[7] Jun Le Goh, *Some computability-theoretic reductions between principles around* ATR_0, Available at https://arxiv.org/abs/1905.06868

[8] Arno Pauly, *An update on Weihrauch complexity, and some open questions*, Available at https://arxiv.org/pdf/2008.11168

▸ A. CORDÓN–FRANCO, F.F. LARA–MARTÍN AND MANUEL J.S. LOUREIRO, *On determinacy of Lipschitz and Wadge games in second order arithmetic*.

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We present a detailed formalization of Lipschitz and Wadge games in the context of second order arithmetic (SOA) and we investigate the logical strength of Lipschitz and Wadge determinacy, and the tightly related Semi-Linear Ordering principle. We show that the topological analysis of the complete sets in Hausdorff difference hierarchy (with respect to Wadge reducibility) developed in [2] can be adapted to prove the determinacy of these games in SOA. As a result, we extend the work developed in [1] and characterize the basic systems from Reverse Mathematics WKL_0, ACA_0 and ATR_0 in terms of these determinacy principles.

Given two formula classes Γ_1 and Γ_2 in the language of SOA, let (Γ_1, Γ_2)-Det_L denote the principle of determinacy for Lipschitz games in the Baire space where player I's pay-off set is Γ_1-definable and player II's pay-off set is Γ_2-definable. A similar principle for Wadge games is introduced and denoted by (Γ_1, Γ_2)-Det_W. Likewise, let (Γ_1, Γ_2)-$\mathsf{SLO}_{L/W}$ denote the corresponding semi-linear ordering principles. If $\Gamma_1 = \Gamma_2 = \Gamma$ then we simply write Γ-$\mathsf{Det}_{L/W}$ or Γ-$\mathsf{SLO}_{L/W}$ and, when restricting ourselves to games in the Cantor space the corresponding principles are denoted by Det^* and SLO^*. Regarding games in the Cantor space we prove that:

1. Over RCA_0, Δ^0_1-Det^*_L and WKL_0 are equivalent.
2. Over RCA_0, Σ^0_1-Det^*_L, $(\Sigma^0_1, \Sigma^0_1 \wedge \Pi^0_1)$-$\mathsf{SLO}^*_{L/W}$ and ACA_0 are pairwise equivalent.
3. Over WKL_0, Σ^0_1-Det^*_W, Σ^0_1-$\mathsf{SLO}^*_{L/W}$ and ACA_0 are pairwise equivalent.
4. Over RCA_0, Δ^0_2-Det^*_L and ATR_0 are equivalent.

As for games in the Baire space we prove that:

1. Over RCA_0, (Δ^0_1, Π^0_1)-Det_L, Π^0_1-Det_L and ATR_0 are pairwise equivalent.
2. Over ACA_0, Δ^0_1-Det_L, Δ^0_1-SLO_L and ATR_0 are pairwise equivalent.
3. Π^1_1-CA_0 proves $(\Sigma^0_1 \wedge \Pi^0_1)$-$\mathsf{Det}_{L/W}$.

(Work partially supported by grant MTM2017-86777-P, Ministerio de Economía, Industria y Competitividad, Spanish Government)

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[2] W. W. Wadge, *Reducibility and Determinateness on the Baire Space*, Ph.D. thesis, University of California, Berkeley, 1983.

▸ KAROL DUDA, AND ALEKSANDER IWANOW, *A finitely presented group with undecidable amenability.*

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We apply the theory of intrinsically computable relations (see [1]) in order to prove the following theorem.

Theorem. *There is a computable group G such that the following problem is undecidable: does a finite subset $F \subset G$ generate an amenable subgroup?*

Applying the method of Clapham (see [2]) we show that the group from the theorem can be made finitely presented.

[1] C. J. Ash and J. F. Knight, *Possible degrees in recursive copies II*, ***Annals of Pure and Applied Logic***, vol. 87 (1997), pp. 151–165.

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▸ MARGARITA GASKOVA, *Boolean algebras autostable relative to n-constructivizations.*
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It is independently proven in [1] and [2] that Boolean algebra is autostable iff it has a finite number of atoms. In [3] it is proved that Boolean algebra is autostable relative to strong constructivizations iff it is isomorphic to a direct product of a finite number of simple models. In this work we study autostability of Boolean algebras with respect to n-constructivizations. Boolean algebra *is autostable relative to n-constructivizations* if it has n-computable representation and any two of it's n-computable representations are computably isomorphic. For $n = 0$, the description is obtained in [1] and [2]. For $n = 1$ and $n = 2$, the description is published by J. B. Remmel in his chapter in [4]. This paper gives a complete description of Boolean algebras autostable relative to n-constructivizations for all natural numbers n.

Theorem. *Let $n \in \omega$. Boolean algebra $\mathcal{A}$ is autostable relative to n-constructivizations iff $\mathcal{A}$ is isomorphic to a direct product of finite number of simple models with elementary characteristics*

- $(i, 1, 0)$, $(i, 0, 1)$ *and* $(j, 1, 0)$, $(j, 0, 1)$, $(j, \infty, 0)$ *for* $j < i$ *if* $n = 4i$.
- $(j, 1, 0)$, $(j, 0, 1)$, $(j, \infty, 0)$ *for* $j \leq i$ *if* $n = 4i + 1$.
- $(i + 1, 1, 0)$ *and* $(j, 1, 0)$, $(j, 0, 1)$, $(j, \infty, 0)$ *for* $j \leq i$ *if* $n = 4i + 2$ *or* $n = 4i + 3$.

Proposition. *Each autostable relative to n-constructivizations Boolean algebra has strong constructivization.*

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► JOSIAH JACOBSEN-GROCOTT, *A characterization of the strongly η-representable many-one degrees.*
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η-representations are a way of coding sets in computable linear orders that were first introduced by Fellner in his thesis. Limitwise monotonic functions have been used to characterize the sets with η-representations, and give characterizations for several variations of η-representations. The one exception is the class of sets with strong η-representations, the only class where the order type of the representation is unique.

We introduce the notion of a connected approximation of a set, a variation on Σ^0_2 approximations. We use connected approximations to give a characterization of the many-one degrees of sets with strong η-representations as well new characterizations of the variations of η-representations with known characterizations.

▸ KATARZYNA W. KOWALIK, *Classifying Ramsey-theoretic principles with strongly infinite witnesses over* RCA_0^*.

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We study the strength of some Ramsey-theoretic statements of the form 'for every infinite set X there exists an infinite set Y such that $\phi(X,Y)$', where $\phi(X,Y)$ is arithmetical. Our base theory is RCA_0^*, which states the existence of computable sets of natural numbers and allows mathematical induction only for Δ_1^0-formulas. RCA_0^* is weaker than the usual base theory considered in reverse mathematics, RCA_0, which contains induction for Σ_1^0-formulas. The weakening of induction allows for a finer analysis of the principles considered, but at the same time leads to a peculiar phenomenon concerning the notion of infinity. Namely, it is consistent with RCA_0^* that an unbounded set of natural numbers does not contain an n-element subset for some $n \in \mathbb{N}$. As a consequence, there are two possible ways of formalizing our principles, depending on 'how infinite' we want the set Y to be.

We consider the effect of requiring Y to be strongly infinite, in the sense of having an n-element finite subset for each $n \in \mathbb{N}$. In 2013, Yokoyama [2] showed that Ramsey's theorem with such strongly infinite witnesses implies Σ_1^0-induction over RCA_0^*. We show that if we require strongly infinite witnesses for other Ramsey-theoretic principles, they tend to behave in one of two ways: they either imply Σ_1^0-induction as well or remain Π_3^0-conservative over RCA_0^*.

Joint work with Marta Fiori Carones and Leszek Kołodziejczyk [1].

[1] Marta Fiori Carones, Leszek Aleksander Kołodziejczyk, Katarzyna W. Kowalik, *Weaker cousins of Ramsey's theorem over a weak base theory*. In preparation.

[2] Keita Yokoyama, *On the strength of Ramsey's theorem without* Σ_1*-induction*, ***Mathematical Logic Quarterly***, vol. 59 (2013), no. 1-2, pp. 108–111.

▶ AGNIESZKA KOZDĘBA, APOLONIUSZ TYSZKA, *The physical limits of computation inspire an open problem that concerns decidable sets $\mathcal{X} \subseteq \mathbb{N}$ and cannot be formalized in ZFC as it refers to the current knowledge on $\mathcal{X}$.*

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E. Landau's conjecture states that the set $\mathcal{P}_{n^2+1}$ of primes of the form n^2+1 is infinite. $f(1)=2$, $f(2)=4$, $f(n+1)=f(n)!$ for $n \in \{2,3,4,\dots\}$. Φ denotes the statement: $card(\mathcal{P}_{n^2+1}) < \omega \Rightarrow \mathcal{P}_{n^2+1} \subseteq [2, f(7)]$. Some systems $\mathcal{U}, \mathcal{A} \subseteq \{x_i! = x_k : i,k \in \{1,\dots,9\}\} \cup \{x_i \cdot x_j = x_k : i,j,k \in \{1,\dots,9\}\}$ are written. Only $(1,\dots,1)$ and $(f(1),\dots,f(9))$ solve $\mathcal{U}$ in $(\mathbb{N} \setminus \{0\})^9$. Φ is equivalent to the statement: *if at most finitely many tuples $(x_1,\dots,x_9) \in (\mathbb{N} \setminus \{0\})^9$ solve $\mathcal{A}$, then they satisfy $x_1,\dots,x_9 \leqslant f(9)$.* No known set $\mathcal{X} \subseteq \mathbb{N}$ satisfies conditions (1)-(4) and is widely known in number theory or naturally defined. (1) *A known algorithm with no input returns an integer n satisfying $card(\mathcal{X}) < \omega \Rightarrow \mathcal{X} \subseteq (-\infty, n]$.* (2) *A known algorithm for every $k \in \mathbb{N}$ decides whether or not $k \in \mathcal{X}$.* (3) *No known algorithm with no input returns the logical value of the statement $card(\mathcal{X}) = \omega$.* (4) *There are many elements of $\mathcal{X}$ and it is conjectured that $\mathcal{X}$ is infinite.* (5) *$\mathcal{X}$ has the simplest definition among known sets $\mathcal{Y} \subseteq \mathbb{N}$ with the same set of known elements.* Conditions (2)-(5) hold for $\mathcal{X} = \mathcal{P}_{n^2+1}$, condition (1) holds assuming Φ. Conditions (1)-(4) hold for $\mathcal{X} = \{k \in \mathbb{N} : (10^6 < k) \Rightarrow (f(10^6), f(k)) \cap \mathcal{P}_{n^2+1} \neq \emptyset\}$, condition (5) fails. Full text: *URL Address*: `http://arxiv.org/abs/1506.08655`.

▶ YANA MICHAILOVSKAYA, *Computable linear orders enriched by the relations $S^n_{\mathcal{L}}$.*
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In this paper, we consider computable linear orders with some additional relations added to their signature. These relations are denoted as $S^n_{\mathcal{L}}$ and are set as follows:
$S^{2k}_{\mathcal{L}}(x,y) \risingdotseq (x <_{\mathcal{L}} y) \& (|(x,y)_{\mathcal{L}}| = 0 \vee |(x,y)_{\mathcal{L}}| = 2 \vee \ldots \vee |(x,y)_{\mathcal{L}}| = 2k)$,
$S^{2k+1}_{\mathcal{L}}(x,y) \risingdotseq (x <_{\mathcal{L}} y) \& (|(x,y)_{\mathcal{L}}| = 1 \vee |(x,y)_{\mathcal{L}}| = 3 \vee \ldots \vee |(x,y)_{\mathcal{L}}| = 2k+1)$,
here $(x,y)_{\mathcal{L}} = \{z \mid x <_{\mathcal{L}} z <_{\mathcal{L}} y\}$.

Proposition 1. For any $(n+1)$-c.e. set A there is a computable linear order $\mathcal{L}$, ordered by type $Sh(\{2,3,...,n+2\})$ such, that $S^n_{\mathcal{L}} \equiv_T A$.

Corollary 1. There is a computable linear order $\mathcal{L}$ such that the spectrum of the relation $S^n_{\mathcal{L}}$ consists exactly of all $(n+1)$-c.e. degrees.

Proposition 2. Let $\mathcal{L}$ be a computable linear order containing only a finite number of blocks of length $\leq n+1$. Then, from the computability $S^n_{\mathcal{L}}$ follows the computability of $S^0_{\mathcal{L}}$.

Theorem 1. Let the computable linear order $\mathcal{L}$ have infinitely many blocks of length 2 and for an odd n the relation $S^n_{\mathcal{L}}$ is computable. Then there is a computable linear order $\mathcal{R}$ such that $\mathcal{R} \cong \mathcal{L}$, $S^n_{\mathcal{R}}$ is computable, but $S^0_{\mathcal{R}}$ is not computable.

Theorem 2. Let the computable linear order $\mathcal{L}$ have infinitely many blocks of length 2 and for an odd n the relation $S^n_{\mathcal{L}}$ is computable. Then there is a computable linear order $\mathcal{R}$ such that $\mathcal{R} \cong \mathcal{L}$, $S^n_{\mathcal{R}}$ is computable, but $\mathcal{L}$ and $\mathcal{R}$ are not computably isomorphic, that is, the computable linear order $(\mathcal{L}, S^n_{\mathcal{L}})$ is not computably categorical.

Corollary 2. A computable linear order $(\mathcal{L}, S^1_{\mathcal{L}})$ is computably categorical if and only if $\mathcal{L} \in \Delta(\{k \cdot \eta : k < \omega\} \cup \{\omega, \omega*, \omega + \omega*\})$ and $\mathcal{L}$ contains only a finite number of blocks of length 2.

Theorem 3. Let $\mathcal{L}$ be a computable linear order in which the following condition is met: for any element $x \in L$, there are elements $x_0 <_{\mathcal{L}} ... <_{\mathcal{L}} x_{n+1} \in L$ such that $S^n_{\mathcal{L}}(x_0, x_{n+1})$ is true and $x <_{\mathcal{L}} x_0$. Let A be a non-computable $(n+1)$-c.e. set such, that $S^n_{\mathcal{L}} \leq_T A$. Then there is a computable linear order $\mathcal{M} \cong \mathcal{L}$ such, that $S^n_{\mathcal{M}} \geq_T A$.

Theorem 4. If a computable linear order $\mathcal{L}$ is not η-like, then the spectrum of the relation $S^n_{\mathcal{L}}$ is closed upwards in c.e. degrees.

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[3] Hirschfeldt D. R., *Degree spectra of intrinsically c.e. relations*, ***Journal of Symbolic Logic***, vol. 66 (2001), no. 2, pp. 441–469.
[4] Moses M., *Recursive linear orders with recursive successivities*, ***Annals of Pure and Applied Logic***, vol. 27 (1984), pp. 253–264.

▸ SERGEI OSPICHEV, *About families in Ershov hierarchy without Friedberg numberings*.
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The theory of numberings gives a fruitful approach to studying uniform computations for various families of mathematical objects. Important tools for this approach are special numberings - for example, principal or positive. One of the most studied special cases is numbering without repetitions, or Friedberg numbering. For example, for any level $n > 0$ of Ershov hierarchy there is infinite Σ_n^{-1}-computable family without Σ_n^{-1}-computable Friedberg numberings[1]. But what will happen to such a family if we move to higher levels of the hierarchy? We show that this property will hold for higher levels:

Theorem. *For any natural $n > 0$ there is Σ_n^{-1}-computable family $\mathcal{S}$ such that there is no Σ_m^{-1}-computable Friedberg numberings of $\mathcal{S}$ for any $m \leq 2n$ but there is Σ_{2n+1}^{-1}-computable Friedberg numbering of this family.*

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▸ PAWEŁ PŁACZEK, *The PTIME complexity of multiplicative nonassociative bilinear logic.*
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Multiplicative-Additive Linear Logic (MALL) was introduced by Girard [6]. Noncommutative MALL (where product $\otimes$ is noncommutative) is due to Abrusci [1]. This logic, presented as a one-sided (precisely: left-sided) sequent system was studied by Lambek [9] under the name: Classical Bilinear Logic.

We study an analogous system for Nonassociative Bilinear Logic (NBL), being a version of Bilinear Logic with nonassociative $\otimes$. Some related logics, restricted to multiplicative connectives and not admitting multiplicative constant (nor the corresponding unit elements in algebraic models), were studied in [7, 3] under the name: Classical Nonassociative Lambek Calculus (CNL). CNL contains one (cyclic) negation $^\sim$, satisfying $a^{\sim\sim} = a$ in algebras. Buszkowski [4] considers a weaker logic, called Involutive Nonassociatvie Lambek Calculus (InNL), with two negations $^\sim, ^-$, satisfying $a^{-\sim} = a = a^{\sim-}$.

InNL is a conservative extension of Nonassociative Lambek Calculus (NL), due to Lambek [8]; see [7, 3]. It can be shown that NBL is a conservative extension of NL with 1 (NL1). These logics have applications in linguistics as type logics for categorial grammars [9, 7, 3] and seem quite natural from the perspective of modal logics, where $\otimes$ can be regarded as a binary possibility operator.

Here we present one-sided systems of NBL in the language $\otimes, \oplus, ^\sim, ^-, \wedge, \vee, 0, 1$. In our sequent systems, negations appear at variables only (so we consider formulas in negation normal form). Negations of arbitrary formulas are defined in metalanguage. Some systems with negations of formulas in the language were considered in [2] (right-sided) and [5] (two-sided).

The cut–elimination theorem holds for all the presented systems. We prove the decidability of NBL. We show that the multiplicative fragment of NBL (MNBL) is $PTIME$. The algorithm essentialy uses cut elimination. An analogous result for InNL is given in [4].

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► SAM SANDERS, *The Big Six and Big Seven of Reverse Mathematics, and a new zoo.*
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I provide an overview of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable ([2–7]). In particular, we have shown that the following theorems are *hard to prove* relative to the usual scale of (higher-order) comprehension axioms, while the objects claimed to exist by these theorems are similarly *hard to compute*, in the sense of Kleene's S1-S9 schemes ([1]).

1. There is no injection from $\mathbb{R}$ to $\mathbb{N}$.
2. Arzelà's convergence theorem for the Riemann integral.
3. A Riemann integrable function is not everywhere discontinuous (Hankel).
4. Jordan's decomposition theorem.
5. A function of bounded variation on the unit interval has a point of continuity.
6. A lower semi-continuous function on the unit interval attains its minimum.
7. The Bolzano-Weierstrass theorem for countable sets (injections/bijections to)

We show that the final item gives rise to many robust equivalences, which in turn yields the 'Big Six' and 'Big Seven' system of Kohlenbach's higher-order Reverse Mathematics.

We discuss how comprehension is unsuitable as a measure of logical and computational strength in this context; we also provide an alternative, namely a (classically valid) continuity axiom from Brouwer's intuitionistic mathematics.

Finally, our study shows that coding practise common in Reverse Mathematics can significantly change the logical strength of basic theorems pertaining to functions of bounded variation and other 'close to full continuity' notions.

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▸ ANTON SETZER, *A model of computation for single threaded sequential interactive programs*.
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We present two models of computation derived from our formalisation (together with Peter Hancock) of interactive programs in dependent type theory, which define the IO monad using weakly final coalgebras.

The first model covers non-state-dependent interactive programs. An interface consists of commands $\mathrm{C} \in \mathcal{P}(\mathbb{N})$ and responses $\mathrm{R} : \mathrm{C} \to \mathcal{P}(\mathbb{N})$. Examples of commands are the printing of a string with response set a singleton set, or reading input from console with response the string being read. Instructions to actuators and reading from sensors can be represented similarly.

The set of interactive programs for an interface (C, R) is the largest set IO of pairs $\langle c, f\rangle$ with $c \in \mathrm{C}$ and $\{f\} : \mathrm{R}(c) \to \mathrm{IO}$. In order to define a monadic version $\mathrm{IO}(A)$, we add to C termination commands $\mathrm{return}(a)$ for $a \in A$ with $\mathrm{R}(\mathrm{return}(a)) = \emptyset$. One can define monadic composition $\_\!\gg\!\_ : (\mathrm{IO}(A) \times (A \to \mathrm{IO}(B))) \to \mathrm{IO}(B)$.

The second model adds a state to the interface, which determines the set of commands available, and which changes depending on commands and responses issued. So, we have states $\mathrm{S} \in \mathcal{P}(\mathbb{N})$, $\mathrm{C} \in \mathrm{S} \to \mathcal{P}(\mathbb{N})$, $\mathrm{R} \in \prod s \in \mathrm{S}.\mathrm{C}(s) \to \mathcal{P}(\mathbb{N})$ and $\mathrm{next} \in \prod s \in \mathrm{S}. \prod r \in \mathrm{C}(s).\mathrm{R}(s, r) \to \mathrm{S}$. We define $\mathrm{IO}(s)$ as the largest set of pairs $\langle c, f\rangle$ with $c \in \mathrm{C}(s)$ and $\{f\} \in \prod r \in \mathrm{R}(s, r).\mathrm{IO}(\mathrm{next}(s, c, r))$. A monadic version $\mathrm{IO}(s, A)$ for $A \in \mathrm{S} \to \mathcal{P}(\mathbb{N})$ can be defined similarly. Equality is bisimulation.

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▸ PAUL SHAFER, *An infinite* Π_1 *set with no* Δ_2 *cohesive subset*.

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A classic theorem of D. Martin states that there is a co-infinite recursively enumerable set with no maximal superset. By taking complements and appealing to the correspondence between maximal sets and Π_1 cohesive sets, Martin's result may be rephrased as stating that there is an infinite Π_1 set with no Π_1 cohesive subset. We generalize this result by showing that there is an infinite Π_1 set with no Δ_2 cohesive subset. We describe how this generalization naturally arises from recent work on *cohesive powers*, and, time permitting, we sketch a direct proof.

► DONALD STULL, *Algorithmic Randomness and Fractal Geometry*.
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Recent work has shown a deep connection between algorithmic randomness and (classical) fractal geometry. In particular, there is a growing body of research that uses *effective* methods to solve problems in fractal geometry which have, on the surface, nothing to do with computability.

One of the central theorems of factral geometry is Marstrand's projection theorem. Let $E \subseteq \mathbb{R}^2$ be an analytic set. For every angle θ, consider the orthogonal projection of E onto the line making angle θ with the origin. Informally, Marstrand's theorem states that, for almost every angle, this projection has maximal Hausdorff dimension. A natural question is whether the analyticity condition can be weakened, or dropped entirely.

In this paper, we define the notion of *optimal oracles* for subsets $E \subseteq \mathbb{R}^n$. One of the primary motivations of this definition is that, if E has optimal oracles, then the conclusion of Marstrand's projection theorem holds for E. We show that every analytic set has optimal oracles. We also prove that if the Hausdorff and packing dimensions of E agree, then E has optimal oracles. Thus, the existence of optimal oracles subsume the currently known sufficient conditions for Marstrand's theorem to hold.

Under certain assumptions, every set has optimal oracles. However, assuming the axiom of choice and the continuum hypothesis, we construct sets which do not have optimal oracles. This construction naturally leads to a new, algorithmic, proof of Davies theorem on projections.

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In [1], the authors formalize the notion of "first-order part" of a multi-valued function as an operator that takes in input a computational problem f and produces the problem ${}^1f$ with the property that

$${}^1f \equiv_{\mathrm{W}} \max_{\leq_{\mathrm{W}}}\{g \in \mathcal{FO} \,:\, g \leq_{\mathrm{W}} f\},$$

where $\mathcal{FO}$ is the family of multi-valued functions with codomain $\mathbb{N}$. While this notion has been already used (implicitly) in the literature as a useful means to prove separation results, its algebraic properties have not been explored so far.

In this work, we study the interaction between the first-order part and other common operators on multi-valued functions (coproduct, meet, parallel product, compositional product, jump). We also introduce a new operator $(\cdot)^{u*}$, which intuitively corresponds to a finite parallelization where the number of instances is not specified a priori. We show that if $f \equiv_{\mathrm{W}} \widehat{g}$ for some $g \in \mathcal{FO}$ then ${}^1f \equiv_{\mathrm{W}} g^{u*}$. Moreover, we explore the connections between $(\cdot)^{u*}$ and $(\cdot)^{\diamond}$, the latter being an operator introduced in [2] that captures the idea of using a Weihrauch problem as an oracle in a computation.

We show how these results can be used to easily characterize the first-order part of known computational problems.

This is joint work with Giovanni Soldà.

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▸ ANDREY FROLOV, MAXIM ZUBKOV, *Spectral universality of linear orders with one binary relation.*

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The set of Turing degrees relative to which a given algebraic structure $\mathcal{A}$ is computably representable is called the degree spectrum of this structure and is denoted by $dgSp(\mathcal{A})$. The question of describing the degree spectra of algebraic structures is one of the fundamental questions in the theory of computable structures and their models. A. Slinko, D. Hirschfeldt, B. Khusainov, R. Shor [**?**] called a class of structures spectrally universal if any possible degree spectrum of an algebraic structure is realized by a degree spectrum of a structure from this class. They also established the spectral universality of a number of classical classes of algebraic structures, for example, classes such as undirected graphs, lattices, commutative semigroups, and others.

The class of countable linear orders is one of the most difficult in terms of describing the spectra of degrees of all representatives of this class. It is not spectrally universal. This follows, for example, from the fact that the spectrum of degrees of linear order, in contrast to graphs, can form an upper cone of degrees if and only if it contains a computable degree (L. Richter [**?**]). And it is still not known whether there is a linear order whose degree spectrum contains exactly all non-zero degrees.

R. Miller and V. Harizanova [**?**] proved the result of L. Richter for an arbitrary linear order with an additional unary predicate. Thus, the class of structures that are linear orders whose signature is enriched in a unary relation is not spectrally universal.

In this paper, we prove that the binary (and therefore n-ary for any $n \geq 2$) relation on $\mathbb{Q}$ (the natural ordering of the set of rational numbers) is spectrally universal. Namely, it is shown that for any graph there exists a binary relation on $\mathbb{Q}$, whose spectrum coincides with the spectrum of degrees of the graph.

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GEN General

Contributed talks

▸ PAVEL ARAZIM, *Logic not being serious.*
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Logical monism, logical pluralism and even logical nihilism share one basic feature, namely that they reckon with the notion of a logic being right. There is a common supposition that it makes some sense to look for right logic or logics. Yet it is far from clear that it is a good idea and that we should measure logical systems by it. It could be a least refreshing to look for an understanding of logical systems which places their worth precisely in how they do not fully correspond to something they could be considered as portraying, for example argumentation in natural language. I thus propose to see them as specific games and I mean this in two senses. The first one is familiar since Wittgenstein, namely that a logical system can be seen as language game constituted by rules of playing it. It can be complemented, though, by another sense due to Eugen Fink. Fink explicates in what sense playing is unserious, a mere playing. By playing any game, we gain a distance from reality which, on the other hand, enables us to enter into a special kind of relationship with it. Namely, we can see the world somehow as a whole, the world can be glanced at in the game. By contrast, in normal serious business we tend to fall short of seeing the world, being beholden merely to what it is filled with. Maybe logical systems offer us a glance at the practice of argumentation and reasoning as whole? Obviously, as our glance of the world changes how we behave towards the entities we encounter in it, so an encounter with theoretical logic changes how we treat our rationality.

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▸ BRUNO BENTZEN, *How do we intuit mathematical constructions?*.
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There is hardly any doubt that the construction of objects by means of effective rules is an integral part of our mathematical experiences. But is there any sense in which intuition grants us epistemic access to such constructions? When we ordinarily speak of a particular construction of a number, set, function, tuple, or sequence, among other mathematical entities, we often adopt the view of a "construction" as a finite method for determining a certain mathematical object by means that are generally accepted as humanly computable. It is therefore with good reason that Bishop describes a construction as a person program and refuses to identify them with Turing machines.

My aim in this talk is to argue that we gain intuition of at least some elementary constructions (including numbers, tuples, and booleans) by means of objects of intuition founded on basic acts of the intellect. I thus reject Kant's traditional view that intuition is an immediate kind of sensory objectual representation and side with Husserl in viewing intuition as a kind of intellectual perception. I will however draw mainly on ideas advanced by Brouwer and Tieszen on how our intuitions can be mediated by acts of perception, pairing, reflection, and rule-giving and rule-following. The view I propose is that certain constructions are intuited as abstract points through imaginative variations of perceived objects into representations of abstract points, while others are intuited as abstract pairs through reflection on the pairing act of an abstract point with one or more given objects of intuition.

▸ NICK BEZHANISHVILI AND FAN YANG, *Intermediate logics in the team semantics setting.*

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In this work, we explore intermediate logics in the team semantics setting. Team semantics was introduced by Hodges [3], and later advanced by Väänänen in dependence logic [6], and adopted independently in inquisitive logic [2]. Both dependence and inquisitive logic were introduced as extensions of classical logic. Recently several authors have defined different intuitionistic logic-based dependence/inquisitive logic [5, 1, 4]. Our starting point is [1]. The key idea of (intuitionistic) propositional team semantics is that formulas are evaluated in (intuitionistic) Kripke models $\mathfrak{M} = (W, R, V)$ with respect to *sets* $t \subseteq W$ of possible worlds (called *teams*). We also extend the language $[\bot, \wedge, \vee, \to]$ of intuitionistic logic (IPC) with a disjunction $\vee\!\!\!\vee$ on the team level. The system tIPC of the logic in [1] consists of IPC axioms for $[\bot, \wedge, \vee\!\!\!\vee, \to]$, some simple axioms for $\vee$, and the Split axiom $\alpha \to (\phi \vee\!\!\!\vee \psi) \to (\alpha \to \phi) \vee\!\!\!\vee (\alpha \to \psi)$ with $\alpha \in [\bot, \wedge, \vee, \to]$.

We provide two alternative approaches to define intermediate team-based logics, by modifying tIPC with axioms with $\vee$ or $\vee\!\!\!\vee$. Given an intermediate logic $\mathsf{L} = \mathsf{IPC} \oplus \Delta$ with Δ a set of $[\bot, \wedge, \vee, \to]$-axioms, the first approach defines an intermediate logic tL by closing the set $\mathsf{tIPC} \cup \{\alpha(\vec{\beta}/\vec{p}) \mid \alpha \in \Delta,\ \beta_i \in [\bot, \wedge, \vee, \to]\}$ under Modus Ponens. We show that if L is complete for a class F of frames, then tL is also complete for F, provided that L has disjunction property or is canonical. In the second approach we replace the Split axiom of tIPC with other $[\bot, \wedge, \vee\!\!\!\vee, \to]$-axioms. This amounts to changing the underlying structure of teams from $(\wp(W), \supseteq)$ to $(\wp(W), \succcurlyeq)$ with an arbitrary partial order $\succcurlyeq$, and thus generalizing the standard team semantics.

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▶ LUDOVICA CONTI, *Logicality and Abstraction.*

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In this talk, I aim at discussing a criterion recently suggested in order to prove the logicality of the abstraction operators. This criterion – which is called *weak invariance* – presupposes an arbitrary interpretation of the abstraction operators[1] and consists in a generalised version of the Tarskian isomorphism invariance. It turns out to be satisfied, at least on some domains, by all the abstraction operators that index classes of the partitions obtained by invariant equivalence relation (cf. [5]), then, *a fortiori*, by the abstraction relations that exhibit an higher form of invariance and by the (contextually) invariant abstraction principles (cf. [1]).

In the first part of the talk, I will discuss similarities and differences between abstraction operators and other variable-binding operators, like ι, ϵ and η (cf. [5]) – which, for brevity, we will call "choice operators". Firstly, abstraction operators – differently from the choice operators – are not *total*, namely they turn out to be empty whether evaluated in some domains; secondly, logicality (weak invariance) of abstraction operators does not coincide – differently from the logicality of the choice operators – with their purely logical definability (cf. [4]); thirdly, while the logicality of choice operator seems to formalise a property of the whole class of similar expressions, then of the intuitive notion of choice, on the contrary, the logicality of abstraction principles seems to regard only second-order abstraction principles, by excluding, e.g., any first-order abstraction operator and failing to capture the notion of abstraction.

In the last part of the talk, I will compare two schemas of, respectively, second-order and first-order abstraction principles, in order to explore whether some of the limitations mentioned above could be overcome by the adoption of a schematic setting ([3], [5]). On the one side, a schematic second-order abstraction principle (of form $\S(RF) = \S(RG) \leftrightarrow R(F,G)$), where $\S$ is a binary abstraction operator and R any isomorphism invariant equivalence relation, defines an abstraction function from $\wp(\wp(D) \times \wp(D)) \times \wp(D) \to D$ that satisfies the criterion of weak invariance and – differently from the specific unary operators – is total ([5]). On the other side, I will focus a schematic first-order abstraction principle (of form $\S(Ra) = \S(Rb) \leftrightarrow R(a,b)$) – where $\S$ is a binary abstraction operator and R any first-order equivalence relation – and I will inquiry whether also the abstraction function from $\wp(D \times D) \times D \to D$ that it defines could be – differently from the respective unary operators – total and isomorphism invariant.

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[1] Such criterion is available in [2]: given an isomorphism i from a domain D, let i^+ such that for every set γ of objects from D, $i^+(\gamma) = \{i(x) : x \in \gamma\}$. Then, an expression ϕ is invariant just in case, for all domains D, D' and bijections i from D to D', the denotation of ϕ on $D(\phi^D)$ is such that $i^+(\phi^D) = \phi^{D'}$.

▸ AZZA GAYSIN, *H-coloring dichotomy in proof complexity.*
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The $\mathcal{H}$-coloring problem for undirected simple graphs is a computational problem from a huge class of the constraint satisfaction problems (CSP): an $\mathcal{H}$-coloring of a graph $\mathcal{G}$ is a homomorphism from $\mathcal{G}$ to $\mathcal{H}$ and the problem is to decide for fixed $\mathcal{H}$, given $\mathcal{G}$, if a homomorphism exists or not.

The dichotomy theorem for the $\mathcal{H}$-coloring problem was proved by Hell and Nešetřil (1990, J. Comb. Theory Ser. B, 48, 92–110) and it says that for each $\mathcal{H}$ the problem is either p-time decidable or NP-complete. Since negations of unsatisfiable instances of CSP can be expressed as propositional tautologies, it seems to be natural to investigate the proof complexity of CSP.

We show that the decision algorithm in the p-time case of the $\mathcal{H}$-coloring problem can be formalized in a relatively weak theory and that the tautologies expressing the negative instances for such $\mathcal{H}$ have polynomial proofs in propositional proof system $R^*(log)$. To establish this, we use a well-known connection between theories of bounded arithmetic and propositional proof systems.

We complement this result by a lower bound result that holds for many weak proof systems for a special example of NP-complete case of the $\mathcal{H}$-coloring problem, using known results about the proof complexity of the Pigeonhole Principle.

The main goal of our work is to start the development of some of the theory beyond the CSP dichotomy theorem in bounded arithmetic. We aim eventually - in a subsequent work - to formalize in such a theory the soundness of Zhuk's algorithm (2020, J. ACM, 67,1–78), extending the upper bound proved here from undirected simple graphs to the general case of directed graphs in some logical calculi.

▶ MICHAŁ TOMASZ GODZISZEWSKI, LUCA SAN MAURO, *Quotient structures, philosophy of computability theory and computational structuralism.*
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This paper has two main goals. First, we contribute to the large body of work labeled as *structuralism* by both expanding it philosophically and exploring it mathematically. Second, we do the first steps towards a future *philosophy of computability.*

Structuralism is a view in philosophy of mathematics, according to which mathematics is the *general study of structures* and that it does not matter what are the objects instantiating a given structure. The view can be dated back at least to the 1960s, and has been receiving attention since then – including recent discussions. According to this account mathematics is not concerned with the internal nature or specific ontological characterization of the elements in the structure, but with how the elements are related to each other. What demands careful explication here, is then the notion of the structural property. On the level of informal intuitions, structural properties are usually characterized in terms of invariance under structural similarity (which is then usually explicated in terms of the isomorphism of mathematical system instantiating a structure) and abstraction from structurally similar (usually: isomorphic) systems.

Or, in a slogan: *Presentations don't matter.*

Our claim:

- *Computational properties of mathematical systems should be a part of the explication of the notion of structural property;*
- *Some computational property (most notably, "being computable") should count as structural.*

We agree with the structuralist interpretation that the elements of a given mathematical system should not be identified with a particular (e.g. set-theoretic) characterization, but we argue that an important part of the notion of structure is constituted by the computational features that can be exhibited by the systems instantiating the structure.

This requires to:

1. revise the notion of structure in a way that encompasses morphisms, transformations, maps, actions, i.e., everything that belongs to the performative part of mathematics and that cannot be faithfully represented as an object;
2. incorporate computability-theoretic properties of mathematical systems into the explication of the notion of mathematical structure and structural property;
3. give account of the notion of structural similarity that should replace (or at least, expand) the notion of isomorphism in the structuralist interpretation of mathematics;
4. reply to the so-called epistemological access challenge by employing the notion of computation in the structuralist characterization of mathematical discourse and knowledge.

The interpretation we propose has central advantages over what might be called traditional structuralism, especially in giving a fine-grained analysis of structurality and structural similarity and in providing a convincing answer to the epistemological access challenge faced by structuralism (and, noteworthy, other positions in philosophy of mathematics, such as Platonism).

▸ MICHAŁ TOMASZ GODZISZEWSKI, *Fairness and Jutsified Representation in Judgment Aggregation and Belief Merging.*
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Proportional fairness of a voting rule can be characterized as the ability to reflect all shades of political opinion of a society within the winning committee. Recently certain a proportionality property of justified representation(JR) has been defined – intuitively it requires that if there is a group of at least n/k voters whose approval have at least one candidate in common, then it cannot be the case that neither of these voters is represented in the committee. During the talk I will try to demonstrate how we can use the machinery from the field of multiwinner election theory to investigate proportionality properties in the general situation of voting for logical propositions (thus, related logically) in place of candidates only.

▸ MOHAMED KHALED, *Algebras of Concepts and Their Networks: Boolean Algebras.*
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The *network $\mathcal{N}$ of Boolean algebras* is defined to be the graph whose vertices are all Boolean algebras, and which has two types of edges: red edges connecting the isomorphic algebras, and blue edges connecting two Boolean algebras if they are not isomorphic and one of them is a large subalgebra of the other one. A large subalgebra of an algebra $\mathfrak{B}$ is a proper subalgebra that needs only one extra element to generate the whole $\mathfrak{B}$. With the aid of this network, we introduce a notion of distance that conceivably counts the minimum number of "dissimilarities" between two given Boolean algebras; with the possibility that this distance may take the value ∞. See [2].

Viewing Boolean algebras as Lindenbaum–Tarski algebras of some propositional theories, this distance thus counts the minimum number of concepts that distinguish these theories from each other [1, 2]. A connected component of the network $\mathcal{N}$ is a maximal subclass of Boolean algebras with the property that the distance between any two of its members is finite. Thus, the distance between any member of a connected component and an algebra outside this component must be infinite. In this talk, we calculate distances between some special Boolean algebras and we give two interesting examples of connected components of the network $\mathcal{N}$ of Boolean algebras.

[1] M. Khaled, G. Székely, K. Lefever and M. Friend, *Distances Between Formal Theories*, ***The Review of Symbolic Logic***, vol. 13 (2020), no. 3, pp. 633–654.
[2] M. Khaled and G. Székely, *Algebras of Concepts and Their Networks*, ***Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing, vol. 1301*** (T. Allahviranloo, S. Salahshour and N. Arica, editors), Springer, Cham, 2021, pp. 611–622.

▸ JUDIT MADARÁSZ, *Concept algebra of special relativistic spacetime.*
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This is a joint research with H. Andréka, I. Németi, and G. Székely.

We explore the first-order logic conceptual structure of special relativistic spacetime: We describe the algebra of concepts (explicitly definable relations) of Minkowski-spacetime, and draw conclusions such as "the concept of lightlike-separability can be defined from that of timelike-separability by using four variables but not by using three variables", or "no non-trivial equivalence relation can be defined in Minkowski-spacetime" , or "there are no interpretations between the classical (Newtonian) and the relativistic spacetimes, in either direction".

We also show that while the algebras of zero-ary and unary concepts are trivial, two-element ones, the algebra of binary concepts has 16 elements and the algebra of ternary concepts is infinite. These results are true over arbitrary ordered fields as the structure of quantities. Concerning the algebra of concepts over real-closed fields only, the algebra of ternary concepts is atomic, and we give a concrete mathematical description for it. Similar, but different, results are true for classical spacetime and Euclidean geometry. For example, the algebra of binary concepts of classical spacetime has only 8 elements and that of Euclidean geometry has only 4 elements.

Both Leon Henkin and J. Donald Monk expressed the desirability of these kinds of investigations earlier, the above are the first results of this kind.

► JOSÉ M. MÉNDEZ, GEMMA ROBLES, FRANCISCO SALTO, *Three-valued relevance logics.*

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Given a matrix semantics, a conditional is *natural* if the following conditions are fulfilled. (1) It coincides with the classical conditional when restricted to the classical values T and F; (2) it satisfies the *Modus Ponens*; and (3) it is assigned a designated value whenever the antecedent and consequent are assigned the same value. This sense of 'natural' being supposed, the class of all natural 3-valued implicative expansions of Kleene's strong logic is defined in [4]. It developed that a subclass of this class consists of relevance logics in Anderson and Belnap's minimal sense of the term (cf. [1]): they have the 'variable-sharing property'. The aim of this paper is to axiomatize the relevance logics in the aforementioned subclass by leaning upon an overdetermined two-valued Belnap-Dunn semantics (cf., e.g., [2], [3]).

[1] A. R. Anderson, N. D. Belnap, ***Entailment. The Logic of Relevance and Necessity***, Vol I, Princeton University Press, 1975.

[2] N. D. Belnap, *A useful four-valued logic*, ***Modern Uses of Multiple-Valued Logic*** (G. Epstein and J. M. Dunn, editors), D. Reidel Publishing Co., Dordrecht, 1976, pp. 8–37.

[3] J. M. Dunn, *Intuitive semantics for first-degree entailments and "coupled trees"*, ***Philosophical Studies***, vol. 29 (1976), pp. 149–168.

[4] G. Robles, J. M. Méndez, *The class of all natural implicative expansions of Kleene's strong logic functionally equivalent to Łukasiewicz's 3-valued logic Ł3*, ***Journal of Logic, Language and Information***, vol. 29 (2020), no. 3, pp. 349–374.

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▸ IOSIF PETRAKIS, *Chu representations of categories related to constructive mathematics.*
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If $\mathcal{C}$ is a closed symmetric monoidal category, the Chu category $\mathbf{Chu}(\mathcal{C}, \gamma)$ over $\mathcal{C}$ and an object γ of it was defined by Chu in [1], as a $*$-autonomous category generated from $\mathcal{C}$. In [2] Bishop introduced the category of complemented subsets of a set, in order to overcome the problems generated by the use of negation in constructive measure theory. In [4] Shulman mentions that Bishop's complemented subsets correspond roughly to the Chu construction. In this talk, based on [3], we explain this correspondence by showing that there is a Chu representation (a full embedding) of the category of complemented subsets of a set X into $\mathbf{Chu}(\mathbf{Set}, X \times X)$. A Chu representation of the category of Bishop spaces into $\mathbf{Chu}(\mathbf{Set}, \mathbb{R})$ is shown, as the constructive analogue to the standard Chu representation of the category of topological spaces into $\mathbf{Chu}(\mathbf{Set}, 2)$. In order to represent the category of predicates (with objects pairs (X, A), where A is a subset of X, and the category of complemented predicates (with objects pairs (X, A), where A is a complemented subset of X, we generalise the Chu construction on a cartesian closed category by defining the Chu category over a cartesian closed category $\mathcal{C}$ and an endofunctor on $\mathcal{C}$. Finally, we introduce the antiparallel Grothendieck construction over a product category and a contravariant $\mathbf{Set}$-valued functor on it, of which the Chu construction is a special case, if $\mathcal{C}$ is a locally small, cartesian closed category.

[1] M. Barr, ****-Autonomous Categories***, LNM 752, Springer-Verlag, 1979.

[2] E. Bishop, ***Foundations of Constructive Analysis***, McGraw-Hill, 1967.

[3] I. Petrakis, *Chu representations of categories related to constructive mathematics*, ***arXiv:2106.01878v1*** (2021).

[4] M. Shulman, *Linear Logic for Constructive Mathematics*, ***arXiv:1805.07518v1*** (2018).

▸ PAULA QUINON, *The anti-mechanist argument based on Gödel's Incompleteness Theorems, indescribability of the concept of natural number and deviant encodings.*
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This paper reassesses the criticism of the Lucas-Penrose anti-mechanist argument, based on Gödel's incompleteness theorems, as formulated by Krajewski (2020): this argument only works with the additional extra-formal assumption that "the human mind is consistent". Krajewski argues that this assumption cannot be formalized, and therefore that the anti-mechanist argument – which requires the formalization of the whole reasoning process – fails to establish that the human mind is not mechanistic. A similar situation occurs with a corollary to the argument, that the human mind allegedly outperforms machines, because although there is no exhaustive formal definition of natural numbers, mathematicians can successfully work with natural numbers. Again, the corollary requires an extra-formal assumption: "PA is complete" or "the set of all natural numbers exists". I agree that extra-formal assumptions are necessary in order to validate the anti-mechanist argument and its corollary, and that those assumptions are problematic. However, I argue that formalization is possible and the problem is instead the circularity of reasoning that they cause. The human mind does not prove its own consistency, and outperforms the machine, simply by making the assumption "I am consistent". Starting from the analysis of circularity, I propose a way of thinking about the interplay between informal and formal in mathematics.

[1] Gödel, K., *Undecidable Diophantine Propositions*, ***Collected Works, Volume III, Unpublished essays and lectures***, Feferman S., et al. (eds.), Oxford University Press 1995: 164–175.

[2] ——— **1951 is Gödel's 1951 Gibbs lecture. Some basic theorems on the foundations of mathematics and their implications, lecture manuscript*, ***Journal***, ***Collected Works, Volume III, Unpublished essays and lectures***, Feferman S., et al. (eds.), Oxford University Press 1995: 304–323.

[3] Krajewski, S., *On Gödel's Theorem and Mechanism: Inconsistency or Unsoundness is Unavoidable in any Attempt to "Out-Gödel" the Mechanist*, ***Fundamenta Informaticae***, vol. 81 (2007), pp. 173–181.

[4] ——— *On the Anti-Mechnist Arguments Based on Gödel's Theorem*, ***Semiotic Studies***, vol. 34 (2020), no. 1, pp. 9–56.

[5] Lucas, J.R. *Minds, Machines and Gödel* ***Philosophy*** vol. 36 (1961) no. 137, pp. 112-127.

[6] Penrose, R. ***The Emperor's New Mind: Concerning Computers*** Oxford University Press 1989

[7] Quinon P. *The anti-mechanist argument based on Gödel's Incompleteness Theorems, indescribability of the concept of natural number and deviant encodings*, ***Semiotic Studies***, vol. 34(1): 243-266.

[8] Wang, H. ***From Mathematics to Philosophy*** Routledge and Kegan Paul 1974

▸ LORENZO ROSSI, MICHAŁ TOMASZ GODZISZEWSKI, *First-order vs. Second-order theories: searching for deep disagreement.*
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We address the general question: what is the best way to formulate (the foundations of) mathematics? In particular, which kind of theory or theories should we employ, in order to provide axiomatizations and explanations of the foundations of mathematics? Of course, one of the main dichotomies is between first-order foundational theories (such as ZFC plus possible extensions) and second-order theories (whether second-order pure logic or second-order ZF). Here, we explore this question by looking at some of the main technical and conceptual differences between first- and second-order foundational theories, and consider which of the two options offers the best (if any) combination of mathematical, meta-mathematical, and philosophical properties to justifiably carry on foundationalist programs.

One of the main methodological and conceptual guidelines we follow is to keep separate the question of which is the best foundational theory (or theories) from the question of how we justify and explain it. In other words, the theory or theories we select to develop the foundations of mathematics in are to be kept separate from the epistemological theories we use to justify the latter.

Some of the key points of the *deep disagreement* between the Second-Orderist and First-Orderist we reach are with respect to the conception of (mathematical) truth and to the forms of multiversism and/or set-theoretic pluralism.

▸ WILLIAM STAFFORD, *Completeness of intuitionistic logic for generalised proof-theoretic semantics.*

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Proof-theoretic validity offers a justification for the logical laws. However, it has recently been shown that proof-theoretic validity does not offer a semantics for intuitionistic logic, but rather it provides a semantics for intermediate logics which aren't harmonious. This is worrying as baked into the philosophical justification for proof-theoretic validity is the idea that it results in a logic with harmonious rules. I show that the lack of harmony stems from the treatment of atomic sentences, not from the treatment of logical connectives. I propose a modification to proof-theoretic validity that could remove the undue impact of atomic formulas sentences.

▸ RAFFAEL STENZEL, $(\infty, 1)$-*Categorical comprehension schemes.*
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Comprehension schemes arose as a crucial notion in the early work on the foundations of set theory, and hence found expression in a variety of foundational settings for mathematics. In particular, Bénabou ([1]) provided an intuition to define the notion of comprehension schemes for arbitrary fibered categories in a syntax-free way. The notion has been made precise in considerable generality by Johnstone in [2], tieing together the elementary examples given in the glossary of [1] to a structurally well behaved theory.

In this talk – based on [3] – we generalize Johnstone's notion of comprehension schemes to the context of cartesian fibrations over $(\infty, 1)$-categories. In doing so it turns out not only that many results do carry over, but that some pivotal constructions are in fact better behaved for the reason that "evil" meta-mathematical equalities naturally arising in the context of ordinary category theory are implicitly replaced by "good" instances of equivalences between $(\infty, 1)$-categories. Much in the spirit of Univalent Foundations, the study of equality becomes a study of equivalence.

The aim of the talk will be to present some of the central results in [3] and show how to apply them to natural examples arising in higher topos theory as well as higher category theory in general.

[1] J. Bénabou, *Fibered Categories and the Foundations of naive Category Theory*, ***The Journal of Symbolic Logic***, vol. 50 (1985), no. 1, pp. 10–37.

[2] P.T. Johnstone. ***Sketches of an Elephant: A Topos Theory Compendium***, Oxford University Press, Great Clarendon Street, Oxford OX2 6DP, 2002.

[3] R. Stenzel, $(\infty, 1)$-*Categorical comprehension schemes*, ***arXiv:2010.09663***, Preprint, 2020.

▸ ALBERTO TERMINE, FABIO A. D'ASARO, AND GIUSEPPE PRIMIERO, *Modelling depth-bounded Boolean reasoning with Markov decision processes and reinforcement learning.*
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§1. Abstract. It has been widely claimed that classical Boolean logic cannot model real agents with bounded cognitive resources [1]. *Depth-Bounded Boolean Logics* (DBBL) [2] aim to solve this issue by limiting the deductive power of the agent with a bound on the maximum number of "guesses" it can perform. The standard semantic treatment of DBBL, however, offers limited connections with contemporary machine-learning methods. Our ongoing work bridges this gap by offering a different semantic approach to *Depth-Bounded Reasoning* that abstracts away from DBBL and admits it as a special case. The semantics of our language is given in terms of *Markov Decision Processes* (MDPs), thus facilitating the use of *Reinforcement Learning* techniques [3]. The starting point is to define *depth-bounded agents* as stochastic state-transition systems that manipulate Boolean formulae through the application of a finite set of *weighted* rules, the weights representing the rules' cognitive costs. The stochastic "inferential" behavior of depth-bounded agents is modelled by MDPs whose *states* are finite sets of Boolean formulae and *actions* are synctactic rules to manipulate such formulae. Semantically, the depth-bounded entailment relation is modelled as *cost-bounded reachability*, i.e., a given formula ϕ is said n-depth derivable if and only if there is a probability equal to 1 of the agent to reach a ϕ state with an *expected cumulative cost* at most equal to n. Inferences, eventually, are modelled as MDP *policies*. Notice that, given an n depth-bounded sequence $\Gamma \vDash_n \phi$, there exist typically more than one possible inference deriving ϕ from Γ. The first step becomes hence to find the inference that minimizes the expected cumulative cost earned by the agent, i.e., the *optimal policy*. Once the optimal policy has been determined, the derivability of a given depth-bounded sequence $\Gamma \vDash_n \phi$ is verified by checking the reward-bounded reachability of ϕ with respect to that optimal policy.

[1] D'Agostino, M., *An Informational View of Classical Logic*, ***Theoretical Computer Science***, (2015), no 606, pp.79–97

[2] D'Agostino, M., Finger, M., Gabbay, D.M., *Semantics and proof-theory of depth bounded Boolean logics*, ***Theoretical Computer Science***, (2013), no 480, pp.43–68

[3] Sutton, R.S., Barto, A.G., ***Reinforcement Learning - An introduction***, Adaptive computation and machine learning, MIT Press 1998.

▸ SARA L. UCKELMAN, *Women in the history of logic: why does it matter who our foremothers are?*.
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In this talk, I will introduce my current book project, *Women in the History of Logic*, and raise and discuss three important methodological questions:

1. What gets to count as "logic" when writing such a history?
2. Who gets to count as a "logician" when writing such a history?
3. Why does it matter who our logical foremothers are?

I will focus on the final question, by showing how understanding the role of women in the history of logic is not (merely) a matter of properly attributing logical developments to the right people, but that by working to understand how these women were involved in the field, and how they have come to be excluded from our understanding of the history of logic, we can understand how it is that women are still being excluded from the current state of logic, and also what we can do about it.

▶ MICHAŁ WALICKI, *Logic of Sentential Predicates*.
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Extending FOL with quantification over all sentences (of the extended language) does not increase expressive power, hence, does not yield any paradoxes. Further extension with sentential predicates (on sentences, not their names/codes), by a form of definitional extension, does not lead to any paradoxes, either. Thus, every FOL theory has a conservative extension with truth predicate axiomatized by the single sentence $\forall\phi(\mathcal{T}\phi \leftrightarrow \phi)$, [2]. Semantics is obtained by extending a presentation of the standard semantics of FOL using kernels of digraphs [3].

Truth predicate is an example of a sentential predicate. Using such, we can say that John claims to be sometimes lying, $J(\neg\forall\phi(J(\phi) \to \phi))$. Reasoning system LSP, extending LK with two rules, allows then to deduce that he indeed does, while if he does not say anything else, that his claim is paradoxical, implying a contradiction.

Following [1], kernel semantics is refined to semikernels, giving a paraconsistent interpretation of such theories, with John's paradox not affecting truth of most other sentences. LSP is complete for this semantics, while extended with (cut) is complete for the explosive semantics of kernels, where John's paradox excludes any model.

[1] M. Walicki, *Resolving infinitary paradoxes*, ***Journal of Symbolic Logic***, 82(2):709–723, 2017.

[2] ———, *Logic of sentential predicates*, 2021, [in preparation, `https://www.ii.uib.no/~michal/LSP.pdf`].

[3] ———, *Extensions in graph normal form*, ***Logic Journal of the IGPL***, 2021, [to appear, `https://doi.org/10.1093/jigpal/jzaa054`].

► JANNIK VIERLING, *The limits of the n-clause calculus.*
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The automation of proof by mathematical induction is a challenging problem that is of paramount importance for computer science. The n-clause calculus addresses this problem by extending the superposition calculus by a cycle detection mechanism [1]. In this talk we will explore the limits of the n-clause calculus. First, we will show that the induction mechanism of the n-clause calculus is not stronger than unnested applications of the $\exists_1^-$ induction rule. After that, we provide an unprovability result for the n-clause calculus by showing the independence of $x + x = x \to x = 0$ from the theory $[T, \exists_1^-(L(T))\text{-IND}^R]$, where T is the base theory of additive arithmetic with the predecessor function p. The independence is obtained by constructing a model M with non-zero idempotents, whose elements are of the form $n^{[b]}$ with $n \in \mathbb{Z}$ and $b \in \{0, 1\}$ such that $b = 0$ implies $n \in \mathbb{N}$. We prove $M \models [T, \exists_1^-(L(T))\text{-IND}^R]$ by showing that for every T-inductive $\exists_1$ formula $\varphi(x)$, there exists an infinite strictly descending sequence $(n_i)_{i\geq 0}$ of integers such that $M \models \varphi(n_i^{[1]})$, for all $i \in \mathbb{Z}$.

[1] Abdelkader Kersani, Nicolas Peltier, *Combining Superposition and Induction: A Practical Realization*, ***Frontiers of Combining Systems – 9th International Symposium*** (Nancy, France), (Pascal Fontaine, Christophe Ringeissen and Renate A. Schmidt, editors), vol. 8152, Springer, 2013, pp. 7–22.

CSL Logic in Cognitive Science and Linguistics

Invited talks

ORGANIZER:
Andrzej Wiśniewski

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Mariusz Urbański, Adam Mickiewicz University Poznań (mariusz.urbanski@amu.edu.pl)

▸ JONATHAN GINZBURG [JOINT WORK WITH ANDY LÜCKING], *Quantifiers as (quasi)-Referential Pluralities*.
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A great insight of Richard Montague's was to use Mostowski's notion of Generalized Quantification to offer a uniform syntax and semantics for both referential and quantification terms. Via subsequent work of Barwise, Cooper, Keenan, van Benthem, Westerstahl and many others this ushered in a golden age of work on natural language quantification. Nonetheless, there are grounds to question whether a semantics based on Generalized Quantification is optimal as an analysis of the meaning of natural language nominal terms, once we consider certain cognitive considerations, for instance how people interact when clarifying the meaning of such terms and the ability to understand such terms independently of predication. Given these problems for a GQ strategy, I will present an alternative approach, based on viewing nominals as structured pluralities and show its application to various natural language phenomena.

▶ MICHAEL KAMINSKI, *Extending the Lambek calculus with classical negation.*
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The Lambek calculus is tightly related to categorial grammars - a family of formalisms for natural language syntax. The categorial grammars can bear only positive information, whereas, as it has been pointed out (independently) by Wojciech Biszkowski and Heinrich Wansing about 25 years ago, negative information is also of importance. In our talk, we present an axiomatization of the non-associative Lambek calculus extended with classical negation and show that the frame semantics with the classical interpretation of negation is sound and complete for this extension.

▶ PIOTR ŁUKOWSKI, *A Proposal for Formalization of Kahneman and Tversky's Thinking Fast and Slow.*
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Keywords: content implication, logic of content, non-Fregean logic, Suszko, fast thinking, slow thinking, Kahneman, Tversky.

Fregean versus non-Fregean paradigm

Frege's Axiom by Suszko:
There are only two references for all sentences: truth and falsehood. All true sentences have one and the same reference, the truth; all false sentences have one and the same reference, the falsehood. An acceptance of Frege's axiom means to semantically reduce of all sentences to two or more objects, the logical values. Thus, there are sentences with completely different meanings but with the same semantic correlate (i.e. their logical value). This approach leads to many paradoxes, including paradoxes of material implication or paradoxes of self-reference.

Example 1 How to interpret the connective of implication appearing in the classical tautology

$$(\alpha \to \beta) \vee (\beta \to \alpha)$$

For any two sentences, at least one implies the other? What does it mean?

Example 2 How to formalize the liar sentence L? Is

$$L \leftrightarrow \neg L$$

a correct expressing of the sense

$$L \text{ says "} L \text{ is false"?}$$

After all, the liar sentence says nothing about equivalence of sentences.

Example 3: The Linda problem
After getting acquainted with the characteristics of a previously unknown Linda, the subjects assessed the conjunctions of two sentences as more likely than that of one of these sentences. Now, that is as *conjunction fallacy.*
Of course, the list of problems is long, and well known. So, let us follow Suszko and reject Frege's axiom, and let us see what comes out of it. Thus, let every sentence has its own semantic corelate, understood as situation or content.

Two non-Fregean logics plus one not standard inference
One of these logics is well known Suszko's SCI, the second is the logic with content implication CCL. However, the logic closest to our thinking seems to be a specific contentual inference. It is seemingly non-monotonic, with empty set of tautologies inference satisfying the rule of implosion: *from contradiction, nothing*, like in our everyday thinking.

Thinking fast and slow
Using the same class of mappings and the same class of models it is possible to define two kinds of inferences. One is like fast and the second like slow thinking, both described by Kahneman and Tversky.

▸ MARIUSZ URBAŃSKI, *Descriptions are not enough and norms are not forever. Formal modelling of human reasoning processes: triggers, methods and results.*
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Since the end of the XX century, certain areas of logic become more and more oriented towards modelling actual cognitive activities of more or less idealized agents. Drawing on enormous achievements brought about by the mathematical turn that started more than a hundred years ago, logic now has come back to its Aristotelian roots as an instrument by which we come to know anything. The re-forged alliance between logic - now well equipped with sophisticated formal tools - and psychology results in more and more substantial developments in studies on human reasoning and problem-solving. To reap the fruits of this alliance we need to be aware that it leads to a shift in focal points of interest of such studies as well as to the expansion of their methodological repertoire. In this talk, I argue that such a practical, or cognitive, turn in logic results in (1) the concept of error becoming crucial for formal modelling of human reasoning processes, (2) prescriptive perspective, which takes into account human limitations in information processing, becoming the most interesting vantage point for such research and (3) triangulation of formal methods, quantitative approach and qualitative analyses becoming the most effective methodology in formal modelling studies.

Contributed talks

▸ JAKUB DAKOWSKI, ALEKSANDRA DRASZEWSKA, BARBARA ADAMSKA, DOMINIKA JUSZCZAK, ŁUKASZ ABRAMOWICZ, ROBERT SZYMAŃSKI,
Addressing logic students' proof making difficulties with Plugin Oriented Programming and gamification.
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There have been several attempts at creating Intelligent Tutoring Systems for several proof methods. Nowadays such software usually can give demonstrations and in some cases finish proofs that were already started [1]. Unfortunately, these tools tend to be pretty limited when it comes to customisation options and overall user experience.

This project, called Larch, aims at improving these aspects of ITS. Since its very beginning, it has been the authors' intention to create an intuitive logical assistant, which would also be usable in didactics. The application guides its users through a chosen proof method, laying out possible options and their consequences, it empowers students to become more proficient, all in a user-friendly way. To achieve that, techniques of gamification were incorporated into the software, along with the best UX practices. Plugin Oriented Programming paradigm was used to create a versatile system, adaptable to users' needs. The most notable plugins created so far include: an implementation of analytic tableaux for propositional logic, sequent calculi for intuitionistic logic with loop prevention mechanism, and a script for generating TeX code. Long-term development plans include implementing other plugins as well as measuring the effectiveness of Larch as a tutoring application for students.

[1] Cristiano Galafassi, Fabiane F.P. Galafassi, Eliseo B. Reategui, Rosa M. Vicari, *EvoLogic: Intelligent Tutoring System to Teach Logic*, ***Brazilian Conference on Intelligent Systems*** (Rio Grande, Brazil), (Ricardo Cerri, Ronaldo C. Prati, editors), vol. 1, Springer, 2020, pp. 110–121.

▸ ANDRZEJ GAJDA, *Abductive reasoning in a neural-symbolic system.*
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In my talk I am going to describe how abductive reasoning can be modelled in a neural-symbolic system, and present implementation of the whole procedure along with results obtained for chosen abductive problems. The definition of abductive reasoning that I am using is worded from algorithmic perspective given by Gabbay and Woods [1], where an abductive hypothesis is an additional information added to the initial knowledge base, that makes it possible to derive (or prove) a formula that was not derivable from that knowledge base as it was initially structured. The neural-symbolic system used in this work is one given by Garcez et al. [2], which allows to translate logic programs into artificial neural networks that can be trained by means of backpropagation algorithm, and translate artificial neural networks into logic programs. The initial knowledge base and abductive problem are worded in logic programs language and then translated into a neural network. The abductive procedure generates abductive hypotheses in the process of neural network training and then translating trained neural network into a logic program. I will also show that this abductive procedure generates abductive hypotheses that fulfill certain criteria [3], like for example consistency with the knowledge base, minimalism or inability of deriving abductive goal from the abductive hypothesis alone.

[1] Dov M. Gabbay and John Woods, ***The Reach of Abduction. Insight and Trial***, Elsevier, London, 2005.

[2] Artur S. d'Avila Garcez, Krysia Broda, and Dov M. Gabbay, ***Neural-Symbolic Learning Systems: Foundations and Applications***, Springer-Verlag, London, 2002.

[3] Maciej Komosinski, Adam Kups, Dorota Leszczyńska-Jasion, and Mariusz Urbański, *Identifying efficient abductive hypotheses using multi-criteria dominance relation*, ***ACM Transactions on Computational Logic, 15(4), 2014.***, vol. 15 (2014), no. 4.

▸ PAWEŁ ŁUPKOWSKI, *Erotetic search scenarios and blackboard architecture in group question decomposition.*
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In the paper I present how erotetic search scenarios (ESS)—a tool developed within a framework of the Inferential Erotetic Logic (IEL)—may be used to model a decomposition of a complex problem into simpler sub-problems. Such an initial problem is represented as the initial question of ESS, and the decomposition process results in a series of auxiliary questions obtained in a systematic manner. ESS has a tree-like structure with the main question as the root and direct answers to it as leaves. Other questions are auxiliary.

I present how ESS may be used to model a question decomposition by a single agent. In what follows, I propose a method relying on the blackboard architecture which allows for modeling a decomposition of a complex question by a group of agents. The aim is to express a situation where agents solve a complex problem in the cooperative manner. The central element is the blackboard visible for all the agents. We have one main agent, called the Writer (who writes down questions and information on the blackboard) and other agents involved in the problem-solving process. As for agents from the group we assume that they have different knowledge concerning the problem in question. We also assume that the group deals with a complex question which cannot be resolved by any agent individually. This question is then written down on the blackboard along with the common knowledge of the group. Afterwards the initial group question is decomposed on a group level into a series of simpler questions (using the aforementioned common knowledge). In what follows, these simpler questions are analyzed by group members (at this level ESSs for each agent are introduced). The last step is collecting the solutions to these auxiliary questions by the Writer and establishing the answer to the initial question.

▸ ALBA MASSOLO, INÉS CRESPO, *Arguments against a Bayesian approach to the normativity of argumentation.*
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Corner & Hahn [1] argue in favor of a Bayesian grounding of normative standards for rational argumentation. We wish to take issue with this strategy, attacking two different angles.

Corner & Hahn find support in [2], but this sort of study presupposes logical monism, while in the past decades logical pluralism has become a strong position in the philosophy of logic [2] [3]. Assuming a contextual logical pluralism, we argue in favor of an externalist characterization of the normativity of logic, where practices themselves are to be seen as sources of normative standards for rational argumentation. Besides, Corner & Hahn's endorsement of a Bayesian account assumes that rational argumentation is only, or mostly, evidence-based reasoning. However, this model seems inadequate if one considers different contexts of argumentative practices, as is the case for mathematics.

Corner & Hahn claim that intuitions about argument strength, or logical validity, match the adequacy of Bayesian formalization as providing normative standards for rational argumentation. However, this match doesn't show that those intuitions play any rôle as normative standards. Furthermore, one should wonder whether anyone's intuitions count. Resnik [5] claims that only expert's intuitions count when it comes to fixing the reflective equilibrium issued by inferential practices. By contrast, we argue that one can see normative standards be issued, not by individual's intuitions, but rather by the argumentative practices which take place within different communities.

[1] A. Corner & U Hahn, *Normative theories of argumentation: Are some norms better than others?*, ***Synthese***, 190 (16), 3579-3510, 2013.

[2] M. Oaksford & N. Chater, *A rational analysis of the selection task as optimal data selection*, ***Psychological Review***, 101, 608–631, 1994

[3] JC Beall & G. Restall, ***Logical Pluralism***, Name of series, Oxford, Oxford University Press, 2006.

[4] C. Caret, *Why logical pluralism?* , ***Synthese***, https://doi.org/10.1007/s11229-019- 02132- w, 2019.

[5] M. Resnik, *Consequence and Normative Guidance*, ***Logic: normative or descriptive? The ethics of belief or a branch of psychology?***, 52, 221-238, 1985.

▶ LACHLAN MCPHEAT, MEHRNOOSH SADRZADEH, HADI WAZNI,, GIJS WIJNHOLDS, *Vectorial discourse analysis in Lambek calculus with a bounded relevant modality.*
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Lambek calculus, the Gentzen-style sequent calculus of a residuated non-commutative monoid, provides a logic for the syntactic structure of preliminary fragment of natural language. Categorial grammarians such as Moortgat and Morrill showed how adding extra operators to the calculus makes it applicable to fragments witnessing phenomena that involve a form of movement, as in relative clauses. More recently, [1] showed that adding a relevant modality makes the calculus applicable to the notoriously complex phenomena of parasitic gaps. In previous work [2], we developed a categorical vector space semantics for this calculus where modal types were interpreted as Fock spaces and copying was obtained via a Frobenius comultiplication. We later showed that the calculus can also be applied to co-reference resolution and can distinguish between the strict /sloppy readings of ambiguous sentences. That framework faced two problems. Firstly, the calculus was undecidable. Secondly, the Frobenius comultiplication only provided approximations of the desired vector copying. In this paper, we redo all the previous work for the newly developed Soft Subexponentials of Lambek calculus [3], which is decidable. We show how the required full copying operation is now obtainable via the layer-wise projections of bounded Fock spaces. We implement the constructions on a large scale corpus, build vector semantics for datasets of parasitic gap noun phrases and elliptic sentences and show how our constructions advance the natural language processing tasks of disambiguation and similarity.

[1] Kanovich, Kuznetsov, Scedrov. Undecidability of the Lambek calculus with a relevant modality. Lecture Notes in Computer Science, 9804:240–256, 2016.

[2] McPheat, Sadrzadeh, Wazni, Wijnholds. Categorical vector space semantics for Lambek calculus with a relevant modality, Electronic Proceedings in Theoretical Computer Science, 333:168–182, 2021.

[3] Kanovich, Kuznetsov, Nigam, Scedrov. Soft subexponentials and multiplexing. Lecture Notes in Computer Science, 12166: 500-517, 2020.

▸ FRANCISCO SALTO, CARMEN REQUENA, *Brain activity marks of logical validity: Results from EEG and MEG studies.*

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The objective of this research is to verify or refute the presence of specific cerebral electrical marks in logically valid inferences. The studies focus on extensional bivalent truth-functional inferences in which logical validity (in classical sense) and probabilistic p-validity (in Adam's sense) coincide. Remarkably, it is not presupposed that any inference exemplifying a deductive argument is eo ipso a deductive inference. This has been an a priori assumption in cognitive neuroscience and it is important because, if understood as abstract relations among propositions or probabilities [2], deductive arguments are clearly distinct from non-deductive arguments. However, as time-consuming cortical events, deductive inferences are far less clearly distinct from non-deductive inferential processes [1]. 23 subjects in the MEG study and 20 in the EEG research were placed into a two conditions paradigm framed in the SET game, with 100 trials for each condition, logically valid vs invalid [3]. Results show: (i) deductive inferences with the same content evoke the same electromagnetic response pattern in both logically valid and invalid conditions, (ii) the amplitude and intensity is lower in valid deductions, significantly in the MEG study (p=0.0003), (iii) reaction time in valid deductions was significantly higher (54.37% in MEG and 61.54% in EEG), (iv) time/frequency patterns of valid deductions show beta-2 band activations at early (300ms) and late (650ms) stages (p-value 0.005), (vi) valid deductions involve frontal connectivity patterns and bands dynamically distinct from invalid inferences. As a conclusion, validity leaves a measurable electrical trait in brain processing. Valid inference is a less-demanding and slow automatism, probably attributable to the recursive and automatable character of valid deductions, suggesting a physical indicator of computational deductive properties.

[1] A. Chuderski, *The relational integration task explains fluid reasoning above and beyond other working memory tasks*, ***Memory & Cognition***, vol. 42 (2014), no. 3, pp. 448–463.

[2] G. Harman, *The relational integration task explains fluid reasoning above and beyond other working memory tasks*, ***Foundations: Logic, Language and Mathematics***, (H. Leblanc and E. Mendelson and A. Orenstein, editors), Springer, New York, 1984, pp. 107–127.

[3] F. Salto, C. Requena, P. Álvarez-Merino, L. Antón, F. Maestú, *Brain electrical traits of logical validity*, ***Scientific Reports***, vol. 11 (2021), no. 7982, pp. 1–13.

▶ URSZULA WYBRANIEC-SKARDOWSKA, *A formal-logic approach to the ontology of language.*

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The ontology of language is understood here as a general formal-logical theory of language, considered as a particular ontological being and generated by the classical categorial grammar. The main goal of this paper is to outline the theory in accordance with the logical conception of language proposed by K. Ajdukiewicz [1] and formalized on the basis of classical logic and set theory. The theory is sketched with respect to the dual ontological status of linguistic expressions as either *concreta* – i.e. *tokens*, in the sense of material, physical objects – or *types*, in the sense of *classes of tokens* – i.e. abstract, ideal objects. Such a duality takes into account two different levels of formalization of the theory of linguistic syntax, semantics and pragmatics, one stemming from *concreta*, construed as linguistic *tokens* of expressions, the other – from their classes, namely *types*, conceived as abstract beings. The two dual-aspect theoretical approaches to linguistic syntax are logically equivalent. The outcome of the considerations is recognition of complete analogousness between the syntactic notions of the two levels, so logic does not settle which view pertaining to the nature of linguistic objects – the concretistic one or the idealistic, platonizing, one – is correct. The basic semantic-pragmatic notions of 'meaning' and 'denotation' are used only with reference to expressions-*types* of language, but their definitions require using some notions for expressions-*tokens*. Considerations related to the formalization of the categorial language lead to the statement that the logic applied here (using set theory) is ontologically neutral due to the existential assumptions regarding the existence of linguistic expressions and their extra linguistic counterparts.

[1] K. Ajdukiewicz, ***Pragmatic logic***, Synthese Library, vol. 62, Reidel-PWN, Dordrecht-Boston-Warsaw, 1975, p. 12.

[2] U. Wybraniec-Skardowska, *Logic and Ontology of Language*, ***Contemporary Polish Ontology*** (B. Skowron, ed.), De Gruyter, Berlin-Boston, 2020, pp. 109–132.

MEP Modal and Epistemic Logic

Invited talks

Organizer:
Nina Gierasimczuk

Invited speakers:
Alexandru Baltag, University of Amsterdam (thealexandrubaltag@gmail.com)
Thomas Bolander, Technical University of Denmark (tobo@dtu.dk)
Helle Hvid Hansen, University of Groningen (h.h.hansen@rug.nl)
Sophia Knight, University of Minnesota Duluth (sophia.knight@gmail.com)

▶ ALEXANDRU BALTAG, *The Logic of Cantor's Derivative and the Perfect Core: a topo-logical exploration of unknowable worlds, surprise exams and other epistemic paradoxes.*
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What is the epistemic meaning of Cantor- Bendixson's derivative? A common epistemic (mis)interpretation is that Cantor derivative is a good topological model for belief. In several joint papers, I criticized this interpretation as ad-hoc and highly problematic. In this talk, I give what I think is the correct answer to the above question, generalizing to arbitrary topologies an idea that goes back (in a restricted, S5 setting) to an old paper by Rohit Parikh.

I start by briefly reviewing the view of topology as a model for evidential epistemology, in particular recalling the two epistemic interpretations of topological interior: as knowledge, or as knowability (by the corresponding agent), depending on whether the topology is taken to represent "evidence in hand" (=the actual evidence currently available to the agent) or "evidence out there" (=the potential evidence, that might be observed or learnt by the agent). I then move to the derivative modality $D(P)$, extracting its epistemic meaning from that of the interior modality. It turns out that $D(P)$ captures the "lack of knowledge", or "unknowability", of the actual world even in the presence of additional information P.

Once derivative is thus understood, you'll be wondering how did you ever manage to do any epistemic logic without it. I show that derivative and its multi-agent generalizations play a key role in a wide range of well-known epistemic puzzles: from the Wise Men (or Muddy Children) puzzle, to the Two Numbers' Puzzle, to the Surprise Exam Paradox. I explain how the Cantor-Bendixson process of iterating derivatives $D(P)$, $D(D(P))$, etc, models the informational dynamics underlying all these puzzles, and how the (non-)paradoxicality of different scenarios is related to the (non-)emptiness of the greatest fixed point of this process: the "perfect core" $D^{\infty}(P)$ of the set P.

I then present a complete axiomatization of the logic of Cantor's derivative and the perfect core, and prove its decidability. This last part is based on joint work with Nick Bezhanishvili and David Fernandez-Duque.

▸ THOMAS BOLANDER, *From Dynamic Epistemic Logic to Socially Intelligent Robots.*
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Dynamic Epistemic Logic (DEL) can be used as a formalism for agents to represent the mental states of other agents: their beliefs and knowledge, and potentially even their plans and goals. Hence, the logic can be used as a formalism to give agents a Theory of Mind allowing them to take the perspective of other agents. In my research, I have combined DEL with techniques from automated planning in order to describe a theory of what I call Epistemic Planning: planning where agents explicitly reason about the mental states of others. Recently, Lasse Dissing, Nicolai Hermann and I have implemented the framework of epistemic planning on physical robots and applied the implementation to human-robot collaboration scenarios. One of the recurring themes is implicit coordination: how to successfully achieve joint goals in decentralised multi-agent systems without prior negotiation or coordination. The talk will first give an introduction to epistemic planning based on DEL and will then demonstrate its use in human-robot collaboration.

▸ HELLE HVID HANSEN, *Automata Minimisation in Logical Form.*
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Recently, two apparently quite different duality-based approaches to automata minimisation have appeared. One is based on ideas that originated from the controllability-observability duality from systems theory, and the other is based on ideas derived from Stone-type dualities specifically linking coalgebras with algebraic structures derived from modal logics.

In this talk, I will present an abstract framework, based on coalgebraic modal logic, that unifies the two approaches. As in the Stone-duality approach, the algebras are essentially logics for reasoning about automata viewed as coalgebras. By exploiting the ability to pass between coalgebras and algebras via a dual adjunction, and extending this dual adjunction to one between automata, we obtain an abstract minimisation algorithm that has several instances, including the Brzozowski minimisation algorithm of DFAs. Further examples include deterministic Kripke frames based on a Stone-type duality, weighted automata based on the self-duality of semimodules, and topological automata based on Gelfand duality, and alternating automata based on the discrete duality between sets and complete atomic Boolean algebras.

▸ SOPHIA KNIGHT, *Reasoning about agents' knowledge about one another's strategies in Strategy Logic*.
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In this talk I will discuss some new developments in Strategy Logic with imperfect information. Strategy Logic is concerned with agents' strategic abilities in multi-agent systems, and unlike ATL, treats strategies as first-class objects in the logic, independent from the agents. Thus, in imperfect information settings, Strategy Logic raises delicate issues, such as what agents know about one another's strategies. I will describe a new version of Strategy Logic that ensures that agents' strategies are uniform, and allows a formal description of their knowledge about each other's strategies. This talk is on joint work with Bastien Maubert, Aniello Murano, Sasha Rubin, Francesco Belardinelli, and Alessio Lomuscio.

Contributed talks

▸ ALEKSI ANTTILA, MARIA ALONI, AND FAN YANG, *A logic for modelling free choice inference.*
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Free choice (FC) is a natural language phenomenon whereby disjunctive sentences appear to license conjunctive inferences:

You may go to the beach or go to the cinema.

$\rightsquigarrow$You may go the beach and you may go to the cinema.

Aloni [1] proposes a bilateral state-based modal logic (BSML) to account for FC. In state-based modal semantics, formulas are interpreted with respect to sets of possible worlds (*states*) rather than the single worlds employed in standard Kripke semantics. BSML extends classical modal logic with a non-emptiness atom NE. NE allows for the representation of a *pragmatic enrichment* of formulas by the pragmatic principle "avoid stating a contradiction". FC inferences are derived as entailments involving pragmatically enriched formulas.

This talk is based on [2]. We consider an extension of BSML with the inquisitive disjunction (BSMLI). We show that BSMLI is expressively complete for the set of state properties invariant under a type of bisimulation for states. We present a complete natural deduction axiomatization for BSMLI; this is a modal extension of a state-based propositional system from [3]. The key new contribution of the system for BSMLI is the provision of rules governing the interaction of NE and the modal operators.

[1] MARIA ALONI, ***Logic and conversation: the case of free choice***, Preprint, https://semanticsarchive.net/Archive/ThiNmIzM/Aloni21.pdf, 2021.

[2] ALEKSI ANTTILA, ***The logic of free choice. Axiomatizations of state-based modal logics***, Master's thesis, University of Amsterdam, 2021.

[3] FAN YANG AND JOUKO VÄÄNÄNEN, *Propositional team logics*, ***Annals of Pure and Applied Logic***, vol. 168 (2017), no. 7, pp. 1406–1441.

▸ GAIA BELARDINELLI, RASMUS K. RENDSVIG, *Epistemic Planning with Attention as a Bounded Resource.*
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Where information grows abundant, attention becomes scarce. As a result, agents must plan wisely how to allocate their attention in order to achieve epistemic efficiency. Here, we present a framework for multi-agent epistemic planning with attention, based on Dynamic Epistemic Logic (DEL, powerful formalism for epistemic planning [1]). The static part of the framework is composed by an *attention state*: a Kripke model augmented with an *attention function* that assigns to each agent a quantitative attention budget. The budget is spent in the dynamic part to learn formulas from a language for attention and knowledge. The learning dynamics are partly captured by an *attention action*: an action model augmented with a *cost function* and a *questioning function*. The cost function specifies how much attention the agent must spend to learn any given formula; the questioning function specifies what formula each agent is attempting to learn the truth-value of, by paying attention to it. A product update then merges the attention state and action to represent the epistemic changes and the relative attention expenditures.

We identify this framework as a fragment of standard DEL, and consider its plan existence problem [1]: given an (initial) attention state, a finite set of attention actions, and a goal formula, is there a finite sequence of the attention actions applicable to the initial attention state that realizes the goal formula? While in the general case the plan existence problem is undecidable, we show that when attention is required for learning, all instances of the problem are decidable.

[1] Thomas Bolander, Tristan Charrier, Sophie Pinchinat, François Schwarzentruber, *DEL-based epistemic planning: Decidability and complexity*, ***Artificial Intelligence***, vol.287, 103304, 2020.

▸ D. BELLIER, M. BENERECETTI, D. DELLA MONICA, AND F. MOGAVERO, *Good-for-Game QPTL: An Alternating Hodges Semantics.*

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The well-established connection between logic and games is witnessed by the fact that satisfiability of a (first-order) logical formula can be reduced to deciding whether a player has a winning strategy in a zero-sum two-player game. Logic can also be used to reason about coalition-games, by encoding moves of the opposing coalitions by means of existentially and universally quantified variables and by describing the game with a formula over those variables. Deciding who wins the game reduces to deciding whether the resulting sentence is satisfiable. When infinite games are considered, one can make the quantified variables range over (infinite) sequences of moves. This leads to first-order extensions of temporal logics, which predicate over infinite sequences of temporal points, one for each round of the game. In this setting, however, the satisfiability and the game solution problems do not coincide anymore, since the choices of one player at each round may depend on the future choices of the adversary.

Inspired by the work on dependence logics [1–3], we propose a novel semantics, generalizing Hodges' one [4], for a first-order extension of Linear Temporal Logic [5], where functional dependencies among the variables can be restricted so that their current values are independent of the future values of the other variables. This allows us to encode various forms of independence constraints and provide a powerful tool to fine-tune the semantics of the propositional quantifiers. In particular, we discuss a specific instantiation of the semantics that allows one to recover a compositional game-theoretic interpretation of the quantifiers and reconcile the satisfiability and the game solution problems. This semantics leads to 2ExpTime decision procedures for both satisfiability and model-checking, heavily reducing the complexity of the logic when interpreted with the standard semantics.

[1] Jouko Väänänen, ***Dependence Logic: A New Approach to Independence Friendly Logic***, Cambridge, London Mathematical Society Student Texts. 2007.

[2] Samson Abramsky, Juha Kontinen, Jouko Väänänen, Heribert Vollmer, ***Dependence Logic: Theory and Applications***, Birkhäuser, 2016.

[3] A.L. Mann and G. Sandu and M. Sevenster, ***Independence-Friendly Logic - A Game-Theoretic Approach***, Cambridge, 2011.

[4] W. Hodges, *Compositional Semantics for a Language of Imperfect Information*, ***Logic Journal of the Interest Group in Pure and applied Logic***, vol. 5 (1997), no. 4, pp. 539-563.

[5] A. Pnueli, *The Temporal Logic of Programs*, ***18th Annual Symposium on Foundations of Computer Science*** (Providence, RI, USA), IEEE, 1977.

▶ WILLEM CONRADIE AND VALENTIN GORANKO, *Algorithmic correspondence for relevance logics.*

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This work brings together two important areas of active development in non-classical logics, viz. *relevance logics* and *algorithmic correspondence theory.*

The classical correspondence theory of modal logic was developed in the 1970s by van Benthem, Sahlqvist, and others, to establish first-order definability and completeness via canonicity for a wide syntactically defined class of modal axioms, commonly referred to as Sahlqvist - van Benthem formulae. These were later generalised to a range of logics with non-classical propositional base in the works of Gehrke, Venema, Nagahasi, Celani, Jansana and others. Algorithmic correspondence theory, first developed in [**?**], transcends the syntactic approach of the classical correspondence theory by developing algorithmic procedures for computing first-order equivalents and proving canonicity of a considerably wider class of input formulae, including all *inductive formulae* [4]. The first such algorithmic procedure, developed for normal modal logics, was SQEMA [**?**], later generalised to ALBA [2] for logic algebraically captured by classes of normal lattice expansions.

In this work, reported in [3], we develop a variation of ALBA for formulae of relevance logics with semantics over Routley-Meyer frames.The resulting algorithmic procedure PEARL computes first-order correspondents with respect to validity in Routley-Meyer frames. It succeeds, inter alia, on a large class of *inductive relevance formulas*, including almost all axioms for important relevance logics known from the literature and it is currently under implementation.

[1] Willem Conradie, Valentin Goranko and Dimiter Vakarelov, *Algorithmic correspondence and completeness in modal logic. I. The core algorithm SQEMA*, ***Logical Methods in Computer Science***, vol. 2 (2006), no. 1, pp. 1–26.

[2] Willem Conradie and Alessandra Palmigiano, *Algorithmic correspondence and canonicity for non-distributive logics*, ***Annals of Pure and Applied Logic***, vol. 170 (2019), no. 9, pp. 923–974.

[3] Willem Conradie and Valentin Goranko, *Algorithmic Correspondence for Relevance Logics I. The algorithm PEARL*, ***Alasdair Urquhart on Nonclassical and Algebraic Logic and Complexity of Proofs*** (Ivo Düntsch and Edwin Mares, editors), Springer, 2021, pp. 163–209.

[4] Valentin Goranko and Dimiter Vakarelov, *Elementary canonical formulae: extending Sahlqvist's theorem*, ***Annals of Pure and Applied Logic***, vol. 141 (2006), no. 1-2, pp. 180–217.

▸ MEGHDAD GHARI, *A temporal logic of justification and obligation.*
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We combine linear temporal logic (with both past and future modalities) with a deontic version of justification logic (cf. [1]) to provide a framework for reasoning about time and epistemic and normative reasons. In addition to temporal modalities, the resulting logic contains two kinds of justification assertions: epistemic justification assertions $[t]_i\phi$ and deontic justification assertions $[s]_i^{\mathcal{O}}\phi$, which are read respectively as "t is agent i's justification for ϕ" and "t is a reason why ϕ is obligatory for agent i". We present a semantics based on interpreted systems in which the truth condition of epistemic justification assertions is given by neighborhood functions (functions that assign to each state/term pair a set of subsets of possible states) and the truth condition of deontic justification assertions is given by binary accessibility relations R^t, for each term t, on possible states. The use of the neighborhood and relational semantics enables us to define the dual of justification assertions, i.e. $\langle t\rangle_i\,\phi$ and $\langle t\rangle_i^{\mathcal{P}}\phi$, which are read respectively as "t is a reason why ϕ is compatible with agent i's knowledge" and "t is a reason why ϕ is permitted for agent i". We then establish soundness and completeness of an axiom system of the logic with respect to this semantics. Further, we formalize the Protagoras paradox in this logic and present a solution to the paradox, and also briefly discuss Leibniz's solution.

[1] S. Artemov, and M. Fitting, *Justification Logic*, ***The Stanford Encyclopedia of Philosophy***, (Spring 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2021/entries/logic-justification/.

▸ TANELI HUUSKONEN, *Cromulence logic: duty meets preference.*
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In standard deontic logic, duties and permissions are treated in terms of an ideal world, in which all obligations are met. This approach leads to several well-known paradoxes and failures to formalize intuitively clear ideas. We propose a framework to link deontic concepts with preferences in a formal way, thereby resolving many of the paradoxes and shortcomings of standard deontic logic.

Cromulence logic, which is how we call our new logic, can be formulated as an extension of the well-known modal logic **S5** with two new binary modal operators. The operators can be chosen to express obligation and permission under a given condition, from which preference operators (strict and nonstrict) can be derived by stipulating that meeting obligations is preferable to failing them. We can also equally well treat preferences as primary concepts and derive obligations from them under a "your wish is my command" assumption.

▸ DOUGLAS MOORE, *Naturality as Universal Normative Authority in Stoic Logic*.
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To be tractable, every science requires first principles. The special sciences embark from axioms and empirical laws ϕ for their first principles employing a rule-based ethic to deductively arrive at consequent knowledge χ. The construct can be represented schematically by the functor F:

$$F : \phi \to \chi \qquad\qquad \#1$$

At the other limit we find metaphysics, the only science lacking determined genus and thus devoid of *a priori* knowledge. This leads to a *right-side* rationality schematic:

$$G : \chi \leftarrow \phi \qquad\qquad \#2$$

Here, rationality flows in the opposite direction with *a priori* knowledge χ' on the right and the consequent ϕ' on the left. This schematic no longer illustrates a syllogism but its converse, a *cosyllogism* (not to be confused with Peirce's *adjunctions*). For that the cosyllogism be *tractable*, *a priori* knowledge χ' must be formalised in some way. We resort to the only viable normative authority available – naturality.

In mathematics, naturality is colloquially regarded as involving constructs that are free of *ad hoc* subjective choices. Traditional set theory mathematics is ill-equipped to formalise the ethics of naturality. The alternative is Category Theory originally developed "to study functors and natural transformations". Natural transformations can be formalised in the form of naturality squares that commute where two sides are left and right adjoints making up "natural" symmetries – arguably the most ubiquitous and fundamental generic structure underlying mathematics.

In this paper, category theory will be shown to participate in its own natural symmetry with its "right adjoint" complementary opposite providing a natural way of formalising the cosyllogistic logic in #2. The paper then goes on to show that the resulting cosyllogistic "right-side" rationality provides a means of reverse engineering the natural rationality underlying the five indemonstrables of ancient Stoic logic.

▸ YUYA OKAWA, SOHEI IWATA, TAISHI KURAHASHI, *Craig's interpolation and the fixed point properties for sublogics of interpretability logic* **IL**.
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The interpretability logic **IL** is a basic logic for investigating the notion of relative interpretability. De Jongh and Visser proved that the fixed point property (FPP) holds for **IL** [2]. Also, Areces, Hoogland and de Jongh proved that Craig's interpolation property holds for **IL** [1].

In a previous work [3], several sublogics of **IL** are introduced and the modal completeness of twenty sublogics of **IL** are investigated. The weakest logic of them is $\mathbf{IL}^-$ and other logics are obtained by adding some **IL**-provable axioms to $\mathbf{IL}^-$.

In this talk, we discuss the fixed point property and Craig's interpolation property for sublogics of **IL**. Firstly, we completely reveal whether the fixed point property holds for the twenty sublogics of **IL**. The logic $\mathbf{IL}^-(\mathbf{J2}_+, \mathbf{J5})$ is the weakest logic of them having FPP, and $\mathbf{IL}^-(\mathbf{J4}, \mathbf{J5})$ is the weakest logic of them having a newly introduced weaker property ℓFPP. Moreover, we reveal whether Craig's interpolation property holds for the seventeen logics. Secondly, we introduce countably many sublogics of $\mathbf{IL}^-(\mathbf{J2}_+, \mathbf{J5})$ (resp. $\mathbf{IL}^-(\mathbf{J4}, \mathbf{J5})$) having FPP (resp. ℓFPP).

[1] Carlos Areces, Eva Hoogland, Dick de Jongh, *Interpolation, definability and fixed points in interpretability logics*, ***Advances in modal logic, Vol. 2 (Uppsala, 1998)*** (Stanford, CA), CSLI Lecture Notes,vol. 119, CSLI Publ, 2001, pp. 35–58.

[2] Dick de Jongh, Albert Visser, *Explicit fixed points in interpretability logic*, ***Studia Logica***, vol. 50 (1991), pp. 39–50.

[3] Taishi Kurahashi, Yuya Okawa, *Modal completeness of sublogics of the interpretability logic* **IL**, ***Mathematical Logic Quarterly***, accepted.

▶ GIANLUCA GRILLETTI, DAVIDE EMILIO QUADRELLARO, *Topological Semantics for Inquisitive and* `DNA`*-logics*.
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We develop a novel topological semantics for inquisitive propositional logic `inqB` and for the entire class of the so-called `DNA`-logics. Inquisitive logic was introduced by a *state-based* semantics in [3], while `DNA`-logics were introduced and studied from a syntactical point of view in [4], as negative variants of intermediate logics. It was shown in [3] that `inqB` is a `DNA`-logic, as it is the negative variant of the Kreisel-Putnam intermediate logic `KP`. An algebraic and topological semantics for `inqB` was introduced in [2], and later extended to all `DNA`-logics in [1].

In this work, we introduce a topological semantics for `DNA`-logics which is based on Esakia semantics and differs from the one considered in [2]. We define for any `DNA`-logic Λ a topological `DNA`-model as a pair $\mathfrak{M} = (\mathfrak{E}, \mu_\neg)$, where $\mathfrak{E}$ is an Esakia space and $\mu_\neg$ is a valuation of atomic formulas over the set $\mathcal{CUR}(\mathfrak{E})$ of regular, clopen upset of $\mathfrak{E}$. Formulas are then interpreted in the usual way and truth is defined by letting $\mathfrak{M} \vDash^\neg \phi$ if $[\![\phi]\!]^{\mathfrak{M}} = \mathfrak{E}$. Similarly, $\mathfrak{E} \vDash^\neg \phi$ holds if $(\mathfrak{E}, \mu_\neg) \vDash^\neg \phi$ for all regular valuations $\mu_\neg : \mathtt{AT} \to \mathcal{CUR}(\mathfrak{E})$.

It follows from Esakia duality together with the completeness of the algebraic semantics studied in [1] that this topological semantics is sound and complete with respect to every `DNA`-logic Λ. Let $Space(\Lambda) = \{\mathfrak{E} \in \mathbf{Esa} : \mathfrak{E} \vDash^\neg \Lambda\}$ and $Log(\mathcal{C}) = \{\phi \in \mathcal{L} : \mathcal{C} \vDash^\neg \phi\}$, then we have the following theorem.

THEOREM 1. $\phi \in \Lambda \iff Space(\Lambda) \vDash^\neg \phi$ *and* $\mathfrak{E} \in \mathcal{C} \iff \mathfrak{E} \vDash^\neg Log(\mathcal{C})$.

In addition, we study regular subsets in Esakia spaces and characterise what are the Esakia spaces dual to regular Heyting algebras:

THEOREM 2. *Let $\mathfrak{E}$ be an Esakia space and $M_\mathfrak{E}$ the Stone space of its maximal elements, then the map $M : \mathcal{CUR}(\mathfrak{E}) \to \mathcal{C}(M_\mathfrak{E})$ such that $M : V \mapsto V \cap M_\mathfrak{E}$ is an isomorphism of Boolean algebras.*

THEOREM 3. *H is a finite, regular, subdirectly irreducible Heyting algebra if and only if the Esakia space $\mathfrak{E}(H)$ dual to H is a finite, rooted frame such that:*
(i) For all $x, \in \mathfrak{E}(H)$, there are two distinct $y_0, y_1 \in \mathfrak{E}(H)$ such that $x \preceq y_0$ and $x \preceq y_1$.
(ii) For all $x, y \notin M_\mathfrak{E}$, $S(x) \neq S(y)$, where $S(z) = \{y \in \mathfrak{E} : x \leq y\}$.

[1] NICK BEZHANISHVILI AND GIANLUCA GRILLETTI AND DAVIDE EMILIO QUADRELLARO, *An Algebraic Approach to Inquisitive and DNA-Logics*, ***ILLC PP-2020-10***.

[2] NICK BEZHANISHVILI AND GIANLUCA GRILLETTI AND WESLEY H. HOLLIDAY, *Algebraic and Topological Semantics for Inquisitive Logic Via Choice-Free Duality*, ***Logic, Language, Information, and Computation. WoLLIC 2019. Lecture Notes in Computer Science*** (Rosalie Iemhoff and Michael Moortgat and Ruy de Queiroz, editors), vol. 11541, Springer, 2019, 35–52.

[3] IVANO CIARDELLI AND FLORIS ROELOFSEN, *Inquisitive Logic*, ***Journal of Philosophical Logic***, vol. 40 (2011), no. 1, pp. 55–94.

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▶ ERIC RAIDL, *Definable Conditionals*.
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Conditionals 'if A, [then] C' are difficult to analyze. A standard account has however emerged [3]: a conditional $A > C$ is true in the actual world (roughly) if and only if the closest A-worlds are C-worlds. However, recent reflections suggest to strengthen the defining clause by additional conditions. Different approaches argue for different additional conditions ([2], [7], [4], [6], [1]). In this talk, I present a general method to prove completeness results for such strengthened conditionals, as I developed it in [5].

The problem is this: Imagine you have a conditional of the form

- $\varphi \triangleright \psi$ in world w iff closest φ-worlds are ψ-worlds and X.

Suppose that X is also formulated in terms of closeness. One can then rephrase $\varphi \triangleright \psi$ as $(\varphi > \psi) \wedge \chi$, where χ is the expression corresponding to the condition X. The central question is whether known completeness results for $>$ can be used to obtain completeness results for $\triangleright$. The answer is yes and the paper provides a general method: First, redefine $>$ in terms of $\triangleright$. This backtranslation of $\varphi > \psi$ yields a formula α in the language for $\triangleright$. One can then use this backtranslation to translate axioms for $>$ into axioms for $\triangleright$. This is a looking glass which provides a distorted picture of the logic for $>$, in terms of $\triangleright$. The picture is a logic for $\triangleright$. The article applies the method to several conditional constructions.

[1] Vincenzo Crupi and Andrea Iacona, *The Evidential Conditional*, ***Erkenntnis*** (2021). https://doi.org/10.1007/s10670-020-00332-2

[2] David Lewis, *Causation*, ***Journal of Philosophy***, vol. 70, no. 17, pp. 556–567.

[3] David Lewis, ***Counterfactuals***, Oxford, Blackwell, 1973.

[4] Eric Raidl, *Completeness for counter-doxa conditionals – using ranking semantics*, ***The Review of Symbolic Logic***, vol. 12 (2019), no. 4, pp. 861–891.

[5] ——— *Definable Conditionals*, ***Topoi***, vol. 40 (2021), 87–105.

[6] Hans Rott, *Difference-making conditionals and the Relevant Ramsey Test*, ***The Review of Symbolic Logic***, doi:10.1017/S1755020319000674.

[7] Wolfgang Spohn, *Conditionals: A Unifying Ranking-Theoretic Perspective*, ***Philosophers' Imprint***, vol. 15, no. 1, pp. 1–30.

▶ ERIC RAIDL, ANDREA IACONA, VINCENZO CRUPI, *The logic of the evidential conditional*.

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In a recent work, Crupi and Iacona [1] have suggested an account of conditionals – the *evidential account*. The account rests on the idea that a conditional is true just in case its antecedent supports its consequent. The idea that A supports C is spelled out in terms of two conditions. One is the *Ramsey Test* as understood by Stalnaker and Lewis: in the closest possible worlds in which A is true, C must be true as well. The other is the *Reverse Ramsey Test*: in the closest possible worlds in which C is false, A must be false as well. We call *Chrysippus Test* the conjunction of the Ramsey Test and the Reverse Ramsey Test.

The paper implements the Chrysippus test in a possible world semantic and presents a system of conditional logic which we show to be sound and complete for the evidential account. The proof adapts a general method elaborated by Raidl [2]. For this, the following insights are used: the evidential conditional can be defined from a known Lewisean conditional as a conjunctive strengthening of the later. Conversely, and less obviously, the Lewisean conditional is back-definable from the evidential conditional. This is expressed by a translation between the languages of the two conditionals. It is this bridge which allows transferring results from the known Lewisean conditional to the defined conditional, as we show in [3].

[1] Vincenzo Crupi and Andrea Iacona, *The Evidential Conditional*, ***Erkenntnis*** (2021). https://doi.org/10.1007/s10670-020-00332-2

[2] Eric Raidl, *Definable Conditionals*, ***Topoi***, 40 (2021), 87–105.

[3] Eric Raidl, Andrea Iacona, Vincenzo Crupi, *The Logic of the Evidential Conditional*, ***Review of Symbolic Logic*** (2021). https://doi.org/10.1017/S1755020321000071

▸ GEMMA ROBLES, *Alternative semantical interpretations of the paraconsistent and paracomplete 4-valued logic PŁ4*.

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The logic PŁ4 is characterized by a modification of the matrix determining Łukasiewicz's 4-valued modal logic Ł (cf. [2]). It is a strong and rich paraconsistent and paracomplete 4-valued logic where necessity and possibility (among other) operators are definable without "Łukasiewicz-type modal paradoxes" being provable (cf. [3]). The logic PŁ4 is introduced in [3], but in [1] it is remarked that De and Omori's logic BD_{+}, Zaitsev's paraconsistent logic FDEP and Beziau's 4-valued logic PM4M are equivalent to PŁ4 (cf. [1] and references therein). The fact that the four systems just quoted have been obtained independently from different motivations seems to suggest that they are four versions of a strong and rich natural logic.

PŁ4 is originally interpreted with a two-valued Belnap-Dunn semantics (cf. [3] and references therein). Nevertheless, the aim of the present paper is to provide still another perspective on PŁ4 by endowing it with both a ternary Routley-Meyer semantics and a binary Routley-semantics together with their respective restriction to the 2 set-up case (cf. [4]).

[1] M. De, H. Omori, *Classical Negation and Expansions of Belnap-Dunn Logic*, ***Studia Logica***, vol. 103 (2015), no. 4, pp. 825–851.

[2] J. Łukasiewicz, *A system of modal logic*, ***The Journal of Computing Systems***, vol. 1 (1953), pp. 111–149.

[3] J. M. Méndez, G. Robles, *A strong and rich 4-valued modal logic without Łukasiewicz-type paradoxes*, ***Logica Universalis***, vol. 9 (2015), no. 4, pp. 501–522.

[4] R. Routley, R. K. Meyer, V. Plumwood, R. T. Brady, ***Relevant Logics and their Rivals***, vol. 1, Ridgeview Publishing Co., Atascadero, CA. 1982.

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▶ YANA RUMENOVA, TINKO TINCHEV, *Undecidability of modal definability: the class of frames with two commuting equivalence relations.*
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Let $\mathcal{K}$ be the class of all relational structures with two commuting equivalence relations and $\mathcal{K}^{\text{fin}}$ be the class of all finite structures from $\mathcal{K}$. Our goal is to study the modal definability of sentences with respect to $\mathcal{K}$ (resp. $\mathcal{K}^{\text{fin}}$). Remind that a sentence A from a first-order language with two binary predicate symbols is modally definable with respect to some class of frames if there is a modal formula φ from the propositional modal language with two unary modalities such that A and φ are valid in the same frames from the class. In this talk we prove the following.

Theorem 1. *The problem of deciding the validity of sentences in $\mathcal{K}$ (resp. $\mathcal{K}^{\text{fin}}$) is reducible to the problem of deciding the modal definability of sentences with respect to $\mathcal{K}$ (resp. $\mathcal{K}^{\text{fin}}$).*

Theorem 2. *The first-order theories of $\mathcal{K}$ and $\mathcal{K}^{\text{fin}}$ are heretitarily undecidable.*

Corollary 3. *The problem of deciding the modal definability of sentences with respect to $\mathcal{K}$ (resp. $\mathcal{K}^{\text{fin}}$) is undecidable.*

MTH Model Theory

Invited talks

ORGANIZER:
Ludomir Newelski

INVITED SPEAKERS:
Daniel Hoffmann, University of Warsaw (daniel.max.hoffmann@gmail.com)
Yatir Halevi, Ben Gurion University of the Negev (yatirh@gmail.com),
Pablo Cubides Kovacics, Heinrich-Heine-Universität Düsseldorf (cubidesk@hhu.de)

▸ DANIEL MAX HOFFMANN, *Kim-independence and ranks.*
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There are two main, quite independent, goals of this talk:

- to describe how the Kim-independence can be induced by its counterpart from the absolute Galois group,
- to present some ranks developed for NSOP_1.

Before coming to the aforementioned points of the talk, I will start with a mild introduction to the class of NSOP_1 theories and provide a summary of the recent results in the NSOP_1. Of course, the list of results will be incomplete and just my personal insight into the subject.

The content of the first goal of this talk is based on the results from [3]. Basically, we generalize there theorems of Zoé Chatzidakis and Nick Ramsey on PAC fields ([1],[4]) to the level of arbitrary PAC substructures of a stable model. We show how the Kim-independence descents from the many sorted structure of the absolute Galois group of a PAC structure to the PAC structure itself.

Second goal of the talk is related to searching for a proper rank for any NSOP_1 theory. There is already a notion of rank intended for NSOP_1 which might be found in [2]. However, there are some questions related to this rank and we would like to propose a slightly different approach to the rank for NSOP_1.

[1] Zoé Chatzidakis *Amalgamation of types in pseudo-algebraically closed fields and applications*, Journal of Mathematical Logic, Vol. 19, No. 2 (2019)

[2] Artem Chernikov, Byunghan Kim, Nicholas Ramsey *Transitivity, lowness, and ranks in $NSOP_1$ theories*, 2020. Available on `https://arxiv.org/abs/2006.10486`.

[3] Daniel Max Hoffmann, Junguk Lee *Co-theory of sorted profinite groups for PAC structures*, 2019. Available on `https://arxiv.org/abs/1905.09748`.

[4] Nick Ramsey *Independence, Amalgamation, and Trees*, PhD dissertation, 2018.

▸ YATIR HALEVI (JOINT WORK WITH ASSAF HASSON AND KOBI PETERZIL), *Definable fields in various dp-minimal fields.*
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The study of definable (or interpretable) fields in various fields has a long history, though with relatively few results, in model theory. For interpretable fields the proof usually relies on elimination of imaginaries in some well understood language.

In this talk we outline a proof that every definable field in a dp-minimal valued field K with generic differentiability of definable functions is definably isomorphic to a finite extension of K. This latter condition holds, e.g., in p-adically closed fields, t-convex fields and algebraically closed valued fields (really in any 1-h-minimal dp-minimal valued field).

If time permits, we will briefly outline a general method for the study of fields interpretable in dp-minimal valued fields satisfying generic differentiability of definable functions, which bypasses elimination of imaginaries. More specifically, we show that in some situations the "interpretable" case reduces (locally) to the "definable" case.

▸ PABLO CUBIDES KOVACSICS, *Beautiful pairs and spaces of definable types.*
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We introduce a general notion of beautiful pairs which encompasses classical results of Poizat in the stable case and of van den Dries-Lewemberg/Pillay in the o-minimal case. We obtain an Ax-Kochen-Ershov type result, showing that beautiful pairs of certain classes of henselian valued fields are essentially controlled by the corresponding beautiful pairs of the value group and residue field. As an application, we infer strict pro-definability of various spaces of definable types. For simplicity, the talk will mainly focus on the case of algebraically closed non-trivially valued fields, where the associated spaces of definable types have a concrete geometric interpretation, e.g., the stable completion introduced by Hrushovski-Loeser, and a model theoretic analogue of the Huber analytification of an algebraic variety.

This is work in progress, joint with Martin Hils and Jinhe Ye.

Contributed talks

▸ CLAUDIO AGOSTINI, AND EUGENIO COLLA, *An algebraic characterization of Ramsey monoids*.
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Carlson's theorem on variable words and Gowers' FIN_k theorem are generalizations of Hindman's theorem that involve a monoid action on a semigroup. In short, they state that for any finite coloring of a semigroup there is an infinite monochromatic "span". They differ in the choice of the monoid. Recently, Solecki in [1] isolated from these two theorems the notion of Ramsey monoid, providing a common generalization of them. Then he proved that an entire class of finite monoids is Ramsey. In this talk, I will present some of the result from a joint work with Eugenio Colla, where we prove a generalization of Solecki's theorem, enlarging the class of monoids that can be proved to be Ramsey and reaching a simple algebraic characterization of Ramsey monoids.

[1] Sławomir Solecki, *Monoid actions and ultrafilter methods in Ramsey theory*, ***Forum of Mathematics, Sigma***, vol. 7 (2019), no. e2.

▸ AIZHAN ALTAYEVA, BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *On algebras of binary formulas for almost ω-categorical weakly o-minimal theories.*

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In [1, 2] algebras of distributions of binary isolating formulas for both countably categorical weakly o-minimal theories and quite o-minimal theories with few countable models were described. Here we describe algebras of distributions of binary isolating formulas for almost ω-categorical weakly o-minimal theories.

DEFINITION 1. [3, 4] Let T be a complete theory, and $p_1(x_1), \dots, p_n(x_n) \in S_1(\emptyset)$. A type $q(x_1, \dots, x_n) \in S_n(\emptyset)$ is said to be a $(p_1, \dots, p_n)$*-type* if $q(x_1, \dots, x_n) \supseteq \bigcup_{i=1}^{n} p_i(x_i)$. The set of all $(p_1, \dots, p_n)$-types of the theory T is denoted by $S_{p_1,\dots,p_n}(T)$. A countable theory T is said to be *almost ω-categorical* if for any types $p_1(x_1), \dots, p_n(x_n) \in S_1(\emptyset)$ there are only finitely many types $q(x_1, \dots, x_n) \in S_{p_1,\dots,p_n}(T)$.

The convexity rank of a formula with one free variable was introduced in [5].

THEOREM 2. *Let T be an almost ω-categorical weakly o-minimal theory, $p, q \in S_1(\emptyset)$ be non-algebraic, $p \not\perp^w q$. Then the algebra $\mathfrak{P}_{\nu(\{p,q\})}$ of binary isolating formulas is generalized commutative iff $RC_{bin}(p) = RC_{bin}(q)$.*

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[1] D.YU. EMELYANOV, B.SH. KULPESHOV, S.V. SUDOPLATOV, *Algebras of distributions for binary formulas in countably categorical weakly o-minimal structures*, ***Algebra and Logic***, vol. 56, No. 1 (2017), pp. 13–36.

[2] D.YU. EMELYANOV, B.SH. KULPESHOV, S.V. SUDOPLATOV, *Algebras of distributions of binary isolating formulas for quite o-minimal theories*, ***Algebra and Logic***, vol. 57, No. 6 (2019), pp. 429–444.

[3] K. IKEDA, A. PILLAY, A. TSUBOI, *On theories having three countable models*, ***Mathematical Logic Quarterly***, vol. 44, issue 2 (1998), pp. 161–166.

[4] S.V. SUDOPLATOV, *Classification of countable models of complete theories*, part 1, Novosibirsk: Novosibirsk State Technical University Publishing House, 2018, ISBN 978-5-7782-3527-4, 326 p.

[5] B.SH. KULPESHOV, *Weakly o-minimal structures and some of their properties*, ***The Journal of Symbolic Logic***, vol. 63 (1998), pp. 1511–1528.

▸ JOHN BALDWIN, *Finer Classification of Strongly minimal sets.*
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E-mail: `jbaldwin@uic.edu`.
URL Address: `http://homepages.math.uic.edu/ jbaldwin/`. This is joint work with Viktor Verbovski. We refine Zilber's trichotomy by studying several variants on definable closure and exploring strongly minimal sets with flat geometries. We find the following classes. 0) acl is trivial; acl is non-trivial but: 1) sdcl (see below) is trivial on independent sets (no commutative binary functions), 2) dcl is trivial on independent sets (no binary functions), 4) definable binary functions exist; e.g. quasigroups, ternary rings.This includes the basic Hrushovski example with any admissible μ ($\delta(B) \leq \mu(A/B)$) [CW12]. In particular no structure in class 1) admits elimination of imaginaries. (Verbovskiy has an example with elimination of imaginaries in an infinite vocabulary). This includes the basic ternary Hrushovski example with any admissible μ ($\delta(B) \leq \mu(A/B)$) [CW12].

To distinguish the classes 0)-4) we introduce several notions. We write G_I ($G_{\{I\}}$) for the group of automorphisms of a model M that fix I pointwise (setwise). For either choice of G, $\mathcal{A}$ is G-normal if it is finite G-invariant and strong in M. Then a is in $\mathrm{dcl}(X)$ ($\mathrm{sdcl}(X)$) if a is fixed by G_X, ($G_{\{X\}}$). We introduce the notion of tree-decomposition of a G-normal subset. Under appropriate conditions on μ, we prove for all G normal sets $\mathcal{A}$,by induction on the height of $\mathcal{A}$, that $\mathrm{dcl}^*(I)$ ($\mathrm{sdcl}^*(I)$) is empty when $G = G_I$, (G_I) and I is independent with $|I| < \omega$. (The $*$ means a depends on all elements of I.) In particular, we show that strongly minimal systems from [CW12] and [BP18] can be found in each classes 1)-4).

[BP18]John T. Baldwin and G. Paolini. Strongly Minimal Steiner Systems I: Existence. Journal of Symbolic Logic Published online by Cambridge University Press: 22 October 2020, pp. 1–15

[CW12]E. Hrushovski Perfect countably infinite Steiner triple systems. *Annals of Pure and Applied Logic*, 62:147–166, 1993.

► DMITRY EMELYANOV, BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *On algebras of binary formulas for partially ordered theories.*

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We consider a generalization, for partially ordered theories, of descriptions for algebras of binary isolating formulas [1] for a series of linearly ordered theories [2, 3, 4], based on dense lower semilattices with Ehrenfeucht theories [5, Example 1.1.1.4].

Using Cayley tables for countably categorical weakly o-minimal theories [2] and quite *o*-minimal theories we explicitly define the classes of commutative monoids $\mathfrak{A}_n$, respectively, $\mathfrak{A}_n^{\rm QR}$, $\mathfrak{A}_n^{\rm QL}$, $\mathfrak{A}_n^I$, of isolating formulas for isolated, respectively, quasirational to the right, quasirational to the left, irrational, 1-types p of quite *o*-minimal partially ordered theories with few countable models, with convexity rank $\mathrm{RC}(p) = n$. For an algebra $\mathfrak{P}_{\nu(p)}$ of binary isolating formulas of 1-type p, we have:

Theorem 1. *Let T be a quite o-minimal partially ordered theory with few countable models, $p \in S_1(\emptyset)$ be a non-algebraic type. Then there exists $n < \omega$ such that:*
(1) *if p is isolated then $\mathfrak{P}_{\nu(p)} \simeq \mathfrak{A}_n$;*
(2) *if p is quasirational to the right (left) then $\mathfrak{P}_{\nu(p)} \simeq \mathfrak{A}_n^{\rm QR}$ ($\mathfrak{P}_{\nu(p)} \simeq \mathfrak{A}_n^{\rm QL}$);*
(3) *if p is irrational then $\mathfrak{P}_{\nu(p)} \simeq \mathfrak{A}_n^I$.*

Corollary 2. *Let T be a quite o-minimal partially ordered theory with few countable models, $p, q \in S_1(\emptyset)$ be non-algebraic types. Then $\mathfrak{P}_{\nu(p)} \simeq \mathfrak{P}_{\nu(q)}$ if and only if $\mathrm{RC}(p) = \mathrm{RC}(q)$ and the types p and q are simultaneously either isolated, or quasirational, or irrational.*

This research has been funded by RFBR (project No. 20-31-90004), by KN MON RK (Grant No. AP08855544), and by SB RAS (project No. 0314-2019-0002).

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[2] D.Yu. Emelyanov, B.Sh. Kulpeshov, S.V. Sudoplatov, *Algebras of distributions for binary formulas in countably categorical weakly o-minimal structures*, ***Algebra and Logic***, Vol. 56, No. 1 (2017), pp. 13–36.

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[5] S.V. Sudoplatov,***Classification of countable models of complete theories***, Novosibirsk, Edition of NSTU, 2018.

▸ CHRISTIAN ESPÍNDOLA, *Categoricity theorems in infinite quantifier languages.*
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We investigate several categoricity phenomena in the model theory of infinite quantifier languages, based on a topos-theoretic approach. For this we prove an omitting types theorem for infinite quantifier logics and use as well a completeness theorem for them.

The main insight is a description of the classifying toposes for saturated models in terms of the double negation topology. Using then a proper class of strongly compact cardinals we recover the property of amalgamation and prove that in fact it follows from certain categoricity assumptions.

[1] Espíndola, C., *Infinitary generalizations of Deligne completeness theorem*, ***The Journal of Symbolic Logic***, vol. 85, issue 3, pp. 1147-1162 (2020).

[2] Dickmann, M., *Large infinitary languages*, ***North-Holland Publishing Company*** (1975).

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[5] Makkai, M. and Reyes, G., ***First order categorical logic***, Lecture Notes in Mathematics, vol. 611 (1977).

▶ FRANCESCO GALLINARO, *Around exponential algebraic closedness.*
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Zilber's quasiminimality conjecture ([2], [3]) predicts that all subsets of the complex numbers that are definable using the language of rings and the exponential function are either countable or cocountable. Building on Zilber's work, Bays and Kirby have proved in [1] that the quasiminimality conjecture would follow from the *exponential algebraic closedness* conjecture, also due to Zilber, which states that all systems of exponential polynomial equations which do not contradict Schanuel's conjecture can be solved in the complex numbers. Similar questions, based on analogues of Schanuel's conjecture, arise in the study of other analytic functions, such as the exponential maps of abelian varieties and the modular j-function. The first part of this talk will focus on these conjectures and the interplay between them, while in the second part some results will be discussed, showing how to solve some classes of systems of equations which have a particularly nice geometry.

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[2] Boris Zilber, *Analytic and pseudo-analytic structure*, ***Lecture Notes in Logic, 19. Logic Colloquium 2000, Paris***, (Rene Cori, Alexander Razborov, Stevo Todorčević, and Carol Wood, editors), Cambridge University Press, 2005, pp.392-408.

[3] Boris Zilber, *Pseudo-exponentiation on algebraically closed fields*, ***Annals of Pure and Applied Logic***, vol.132 (2005), no.1, pp.67-95.

► DAVIT HARUTYUNYAN, *On Some Associative Formula with Functional Variables.*
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A binary groupoid (Q, A) is a non-empty set Q together with a binary operation A. The groupoid $(Q, \cdot)$ is division if for any $a \in Q$ L_a and R_a are surjective mappings. The binary algebra (Q, Σ) is a division algebra if (Q, A) is a division groupoid for any $A \in \Sigma$.

We call a groupoid $(Q, \cdot)$ left-regular if $ca = cb \Rightarrow R_a = R_b$, where $a, b, c \in Q$. Similarly, we define the right-regular groupoid. We call a groupoid regular if it is simultaneously left-regular and rightregular. A binary algebra (Q, Σ) is regular if (Q, A) is a regular groupoid for any $A \in \Sigma$.

THEOREM 1. *Suppose (Q, Σ) is a regular division algebra and for any $A, C \in \Sigma$ there exist $B, D \in \Sigma$ such that one of these identities*

$$A(x, B(y, z)) = C(D(x, y), z), \qquad (1)$$
$$A(x, C(y, z)) = B(D(x, y), z), \qquad (2)$$
$$A(x, D(y, z)) = C(B(x, y), z), \qquad (3)$$
$$D(x, B(y, z)) = C(A(x, y), z), \qquad (4)$$

is true. Then, there exists a group $(Q, \cdot)$ such that (Q, Σ) is epitopic to this group.

THEOREM 2. *Suppose (Q, Σ) is a regular division algebra and for any $A, C \in \Sigma$ there exist $B, D \in \Sigma$ such that identity (1) takes place. Then, there exists a group $(Q, \cdot)$ such that the algebra (Q, Σ) is endolinear over this group.*

▶ VALENTIN GORANKO, RUAAN KELLERMAN, *Approximating trees as coloured linear orders and complete axiomatisations of some classes of trees.*
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We study the first-order theories of some natural classes of coloured trees, including the four classes of trees whose paths have the order type respectively of the natural numbers, the integers, the rationals, and the reals. We develop a technique for approximating a tree as a suitably coloured linear order. We then present the first-order theories of certain classes of coloured linear orders and use them, along with the technique for approximating trees as coloured linear orders, and techniques borrowed from [1], to establish complete axiomatisations of the four classes of trees mentioned above. This talk is based on the work presented in [2].

[1] Kees Doets, *Monadic Π^1_1-Theories of Π^1_1-Properties*, ***Notre Dame Journal of Formal Logic***, vol. 30 (1989), no. 2, pp. 224–240.

[2] Valentin Goranko and Ruaan Kellerman, *Approximating trees as coloured linear orders and complete axiomatisations of some classes of trees*, ***Journal for Symbolic Logic***, to appear.

▶ B.SH. KULPESHOV, *On criterion for binarity of almost ω-categorical weakly o-minimal theories.*

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This lecture concerns the notion of *weak o-minimality* which was initially deeply studied by D. Macpherson, D. Marker and C. Steinhorn in [1]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of M is a union of finitely many convex sets in M. The rank of convexity of a formula with one free variable was introduced in [2].

The following notion was introduced in [3] and investigated in [4]. Let T be a complete theory, and $p_1(x_1), \dots, p_n(x_n) \in S_1(\emptyset)$. A type $q(x_1, \dots, x_n) \in S_n(\emptyset)$ is said to be a $(p_1, \dots, p_n)$-*type* if $q(x_1, \dots, x_n) \supseteq \bigcup_{i=1}^{n} p_i(x_i)$. The set of all $(p_1, \dots, p_n)$-types of the theory T is denoted by $S_{p_1,\dots,p_n}(T)$. A countable theory T is said to be *almost ω-categorical* if for any types $p_1(x_1), \dots, p_n(x_n) \in S_1(\emptyset)$ there are only finitely many types $q(x_1, \dots, x_n) \in S_{p_1,\dots,p_n}(T)$.

Theorem 1. *Let T be an almost ω–categorical weakly o-minimal theory. Then T is binary iff every non-algebraic $p \in S_1(\emptyset)$ has finite convexity rank.*

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[2] B.Sh. Kulpeshov, *Weakly o-minimal structures and some of their properties*, ***The Journal of Symbolic Logic***, vol. 63 (1998), pp. 1511–1528.

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[4] S.V. Sudoplatov, *Classification of countable models of complete theories*, part 1, Novosibirsk: Novosibirsk State Technical University Publishing House, 2018, ISBN 978-5-7782-3527-4, 326 p.

▸ IVAN DI LIBERTI, *Formal model theory and Higher Topology*.
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Motivated by the abstract study of semantics, we study the interaction between topoi, accessible categories with directed colimits and ionads. This theory amounts to a categorification of famous construction from general topology: the Scott topology on a poset and the adjunction between locales and topological spaces. This technology is then used in order to establish syntax-semantics dualities. Among the significant contributions, we provide a logical understanding of ionads that encompasses Makkai ultracategories.

[1] Ivan Di Liberti, ***The Scott adjunction***, PhD thesis, arXiv:2009.07320.

[2] Ivan Di Liberti, ***General facts on the Scott Adjunction***, ArXiv:2009.14023.

[3] Ivan Di Liberti, ***Towards Higher Topology***, ArXiv:2009.14145.

[4] Ivan Di Liberti, ***Formal Model Theory & Higher Topology***, ArXiv:2010.00319.

▸ ADAM MALINOWSKI AND LUDOMIR NEWELSKI, *A few remarks on strongly generic sets*.
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A promising approach to model theory of unstable groups is via the methods of topological dynamics (see eg. [1, 2, 3, 5]). For a group G definable over the empty set in a model M, the space $S_G(M)$ of 1-types extending the formula $x \in G$ is naturally a G-flow. The classical Ellis theorem allows to assign to any such model a particular group, called the Ellis group of $S_G(M)$. The study of Ellis groups in model theory aims to achieve a deeper understanding of the structural properties of the theory of M.

In [4] Newelski established a connection between Ellis groups (or more precisely, minimal ideals containing them) and particular algebras of subsets of G called image algebras. Their uncommon property is that they consist of strongly generic sets, i.e. sets A such that every non-empty Boolean combination of G-translates of A is a generic subset of G. I am going to present some results and constructions related to strongly generic sets.

[1] Krzysztof Krupiński, Ludomir Newelski, Pierre Simon, *Boundedness and absoluteness of some dynamical invariants in model theory*, ***Journal of Mathematical Logic***, vol. 19 (2017), no. 2.

[2] Krzysztof Krupiński, Anand Pillay, *Amenability, definable groups, and automorphism groups*, ***Advances in Mathematics***, vol. 345 (2019), pp. 1253-1299.

[3] Ludomir Newelski, *Topological dynamics of definable group actions*, ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 1, pp. 50–72.

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▸ NURLAN MARKHABATOV, SERGEY SUDOPLATOV, *On closures for partially ordered families of theories.*
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We apply a general approach for closures of families of theories [1, 2] for some special cases of partially ordered families.

Definition [2]. For a family $\mathcal{T}$ of theories in a language Σ and a theory T we put $T \in \mathrm{Cl}_1(\mathcal{T})$ if $T \in \mathcal{T}$, or T is nonempty and $T = \{\varphi \in \mathrm{Sent}(\Sigma) \mid |(\mathcal{T}')_\varphi| \geq \omega\}$ for some $\mathcal{T}' \subseteq \mathcal{T}$. If $\mathcal{T}'$ is fixed then we say that T *belongs to the* Cl_1*-closure of* $\mathcal{T}$ *with respect to* $\mathcal{T}'$, and T is an *accumulation point* of $\mathcal{T}$ with respect to $\mathcal{T}'$.

Theorem 1. [2] *For any linearly* $\subseteq$*-ordered family* $\mathcal{T}$*,* $\mathrm{Cl}_1(\mathcal{T})$ *consists of unions for subfamilies of* $\mathcal{T}$*, and of intersections for countable subfamilies of* $\mathcal{T}$ *ordered by the type* ω^**.*

Theorem 2. *For any partially* $\subseteq$*-ordered family* $\mathcal{T}$ *with finitely many maximal chains,* $\mathrm{Cl}_1(\mathcal{T})$ *consists of unions for unions of chains of* $\mathcal{T}$ *and for intersections of countable chains of* $\mathcal{T}$ *which are ordered by the type* ω^**.*

Theorem 2 can fail for the case of infinitely many maximal chains.

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[1] N.D. Markhabatov, S.V. Sudoplatov, *Topologies, ranks and closures for families of theories. I*, ***Algebra and Logic***, Vol. 59, No. 6 (2020), pp. 649–979.

[2] N.D. Markhabatov, S.V. Sudoplatov, *Topologies, ranks and closures for families of theories. II*, ***Algebra and Logic***, Vol. 60, No. 2 (2021). (to appear)

▸ INESSA PAVLYUK, SERGEY SUDOPLATOV, *On rich properties for the family of theories of abelian groups.*
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We continue to study families of theories of abelian groups [1, 2] describing possibilities for sentences with respect to rich properties following a general approach for links between formulas φ and properties P using the ranks RS_P [3].

Following [3], for a property $P \subseteq \mathcal{T}_\Sigma$, a sentence $\varphi \in \mathrm{Sent}(\Sigma)$ is called P-*generic* if $\mathrm{RS}_P(\varphi) = \mathrm{RS}(P)$, and $\mathrm{ds}_P(\varphi) = \mathrm{ds}(P)$ if $\mathrm{ds}(P)$ is defined.

Let $\overline{\mathcal{TA}}$ be the family of all theories of abelian groups in a language Σ_0. A property $P \subseteq \overline{\mathcal{TA}}$ is called *rich* if $P \cap P' \neq \emptyset$ for each nonempty property $P' = \left(\overline{\mathcal{TA}}\right)_\varphi$ defined by a sentence φ locally describing linear (in)dependence, (in)divisibilities and orders of elements.

THEOREM 1. *A property $P \subseteq \overline{\mathcal{TA}}$ is rich if and only if $\mathrm{Cl}_E(P) = \overline{\mathcal{TA}}$.*

THEOREM 2. $|\{P \subseteq \overline{\mathcal{TA}} \mid P$ *is rich*$\}| = 2^\omega$*, moreover,* $|\{P \subseteq \overline{\mathcal{TA}} \mid P$ *is rich and countable*$\}| = 2^\omega$.

THEOREM 3. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and a rich property $P \subseteq \overline{\mathcal{TA}}$ the following possibilities hold:*
(1) $\mathrm{RS}_P(\varphi) = -1$*, if φ is $\overline{\mathcal{TA}}$-inconsistent;*
(2) $\mathrm{RS}_P(\varphi) = 0$*, if φ is $\overline{\mathcal{TA}}$-consistent and belongs to (finitely many) theories in $\overline{\mathcal{TA}}$ with finite models only;*
(3) $\mathrm{RS}_P(\varphi) = \infty$*, if φ belongs to a theory $T \in \overline{\mathcal{TA}}$ with an infinite model.*

COROLLARY 4. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and rich $P \subseteq \overline{\mathcal{TA}}$ either φ is represented by a disjunction of finitely many sentences φ_i isolating theories $T_i \in \overline{\mathcal{TA}}$ with finite models, or φ is P-generic.*

Notice that the assertions above can fail if $P \subseteq \overline{\mathcal{TA}}$ is not rich.

The study was carried out within the framework of the state contract of the Sobolev Institute of Mathematics (project No. 0314-2019-0002) and the Committee of Science in Education and the Science Ministry of the Republic of Kazakhstan (Grant No. AP08855544).

[1] In.I. Pavlyuk, S.V. Sudoplatov, *Families of theories of abelian groups and their closures*, ***Bulletin of Karaganda University. Series "Mathematics"***, vol. 90 (2018), pp. 72–78.

[2] In.I. Pavlyuk, S.V. Sudoplatov, *Ranks for families of theories of Abelian groups*, ***Bulletin of Irkutsk State University. Series "Mathematics"***, vol. 28 (2019), pp. 95–112.

[3] S.V. Sudoplatov, *Formulas and properties*, arXiv:2104.00468v1 [math.LO], 2021.

► LUCA REGGIO, *Game comonads and homomorphism counting in finite model theory*.
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Game comonads [1, 3] have been recently introduced as a means of relating categorical semantics to finite model theory. They hinge on the idea that model-comparison games should be regarded as semantic constructions in their own right, and yield categorical characterisations of key combinatorial parameters of relational structures. For an axiomatic approach to game comonads and their coalgebras, see [2].

In this talk, we present an approach to homomorphism counting results in finite model theory based on game comonads, which has been obtained in joint work with Anuj Dawar and Tomáš Jakl [4]. The first and best-known homomorphism counting result is Lovász' theorem (1967), stating that two finite relational structures A and B are isomorphic if, and only if, for every finite relational structure C, the number of homomorphisms from C to A is the same as the number of homomorphisms from C to B. We explain how game comonads can be used to provide new uniform proofs of Lovász' theorem and more recent results of Dvořák (2010) and Grohe (2020), as well as a novel homomorphism counting result in modal logic.

[1] Samson Abramsky, Anuj Dawar, Pengming Wang, *The pebbling comonad in finite model theory*, 32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2017, pp. 1–12.

[2] Samson Abramsky, Luca Reggio, *Arboreal categories and resources*, to appear in the proceedings of the 48th International Colloquium on Automata, Languages, and Programming (ICALP), 2021.

[3] Samson Abramsky, Nihil Shah, *Relating Structure and Power: Comonadic semantics for computational resources*, 27th EACSL Annual Conference on Computer Science Logic (CSL), 2018, pp. 2:1–2:17.

[4] Anuj Dawar, Tomáš Jakl, Luca Reggio, *Lovász-type theorems and game comonads*, to appear in the proceedings of the 36th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2021.

▸ ALEXEY RYZHKOV, ALEXEY STUKACHEV, AND MARINA STUKACHEVA, *Approximation spaces over dense linear orders.*

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A series of positive results related to the generalized problem of Yu. L. Ershov on the structure of Σ-degrees of dense linear orders [1, 2, 4] is obtained. In particular, we prove that interval models of temporal logic, as well as finite fragments of approximation spaces generated by interval Boolean algebras, are Σ-definable (effectively interpretable) in hereditarily finite superstructures over dense linear orders. These results are used in the analysis of semantics of verbs in natural languages within the approach in formal semantics proposed by R. Montague [3, 5].

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[4] A.I. Stukachev, ***Effective model theory via the Σ-definability approach***, Lecture Notes in Logic, vol. 41, 164–197, 2013.

[5] A.I. Stukachev, ***Approximation spaces of temporal processes and effectiveness of interval semantics***, In: Distributed Computing and Artificial Intelligence, Special Sessions, 17th International Conference, Advances In Intelligent Systems and Computing, vol. 1242, 53–61, 2021.

▶ GERGELY SZÉKELY, *Conceptual Distance and Algebras of Concepts.*
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The notion of *conceptual distance* has recently been introduced in [1]. This distance measures the minimal number of concepts that are needed to be added or removed from a theory to turn it definitionally equivalent to another one. This distance gives a quantitative method to determine the difference between any two nonequivalent theories, which is also qualitative if we keep track of the concrete concepts distinguishing the two theories in hand. For example, using this distance terminology a surprising result of [3] can be reformulated as: the conceptual distance between special relativity and late classical kinematics is 1, and they differ only in the concept of absolute rest of classical kinematics.

In this talk, we are going to introduce a general notion of distance between any two algebras of the same similarity type called the *generator distance*, see [2]. Then we show that, for any two models having finite (but not necessarily the same) first-order languages, the generator distance between the Lindenbaum–Tarski algebras of these models is the same as the conceptual distance between their first-order logic theories.

This connection between the generator distance of Lindenbaum–Tarski algebras of concepts and the conceptual distance of complete theories can give an effective algebraic method to determine the conceptual distance between arbitrary theories, which seems to be a quite difficult task in general.

[1] M. Khaled, G. Székely, K. Lefever and M. Friend, *Distances Between Formal Theories*, ***The Review of Symbolic Logic***, vol. 13 (2020), no. 3, pp. 633–654.

[2] M. Khaled and G. Székely, *Algebras of Concepts and Their Networks*, ***Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing***, vol. 1301 (T. Allahviranloo, S. Salahshour and N. Arica, editors), Springer, Cham, 2021, pp. 611–622.

[3] K. Lefever, *Using Logical Interpretation and Definitional Equivalence to Compare Classical Kinematics and Special Relativity Theory*, PhD Dissertation, Vrije Universiteit Brussel, 2017.

▶ AIGERIM DAULETIYAROVA, VIKTOR VERBOVSKIY, *On local monotonicity of unary functions definable in o-stable ordered groups.*
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Let $\mathcal{M} = (M, <, \dots)$ be a totally ordered structure. A partition $\langle C, D\rangle$ of M is called a *cut* if $C < D$. Given a cut $\langle C, D\rangle$ one can construct a partial type $\{c < x < d : c \in C,\ d \in D\}$, which we also call a cut and use the same notation $\langle C, D\rangle$. A cut $\langle C, D\rangle$ in an ordered group is called *non-valuational* [2] if $d - c$ converges to 0 whenever c and d converge to $\sup C$ and $\inf D$ accordingly. An ordered group G is said to be of *non-valuational type*, if there is no definable non-trivial convex subgroup in G.

Definition 1 (B. Baizhanov, V. Verbovskiy [1]).

1) An ordered structure $\mathcal{M}$ is *o-stable in* λ if for any $A \subseteq M$ with $|A| \le \lambda$ and for any cut $\langle C, D\rangle$ in $\mathcal{M}$ there are at most λ 1-types over A which are consistent with the cut $\langle C, D\rangle$.

2) A theory T is *o-stable in* λ if every model of T is. A theory T is *o-stable* if there exists an infinite cardinal λ in which T is o-stable.

Here we study o-stable ordered groups, the initial study of them was in [3], [4].

Theorem 2. *Any unary function that is definable in an o-stable ordered group of non-valuational type is piecewise continuous and monotone (note that pieces need not be convex).*

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▸ AIBAT YESHKEYEV, AIGUL ISSAYEVA AND NAZGUL SHAMATAYEVA, *On atomic and algebraically prime definable subsets of semantic model.*
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In current abstract we are giving the result which connected with the different types of atomic and prime models in the frame of Jonsson theories investigations. In first time definable atomic and algebraically prime subsets of semantic model was defined in [1]. Such point of view is a refining of some questions which raised in [2], where relations between atomic and algebraically prime models was studied.

Let us give a necessary definitions.

Definition 1. 1) α-type is called any set of formulas consistent with T, the free variables of which are found in $\bar{x}^\alpha$

2) α-type ρ is called Γ-ω-type, if $\rho \subseteq \Gamma$

3) Γ-ω-type ρ is called Γ_1-principle type, if there exists such a sequence $\langle \psi_n(\bar{x}^n) : 1 \leq n < \omega \rangle$ Γ_1-formulas, such that:

a) $T \cup \psi_n(\bar{x}^n)$ is consistent, $1 \leq n < \omega$;

b) $\psi_n(\bar{x}^n)$ generates $\rho \restriction \bar{x}^n$, where $\rho \restriction \bar{x}^n$ is set of formulas from ρ, the free variables of which are among $(x_1, ..., x_n)$, $1 \leq n < \omega$;

c) $T \vdash \psi_n(\bar{x}^n) \leftrightarrow \exists \bar{x}^{n+1} \psi_{n+1}(\bar{x}^{n+1})$, $1 \leq n < \omega$.

Definition 2. A set A_1 is called fine almost weakly (Γ_1, Γ_2)-*cl*-atomic in the theory T, if

1) every ω sequence of elements A_1 satisfied Γ_1-principle type for Γ_2-ω-type.

2) $cl(A_1) = M_1, M_1 \in E_T$, where E_T is a class of all existentially closed models of the theory T;

and obtained model M_1 is said to be fine almost weakly (Γ_1, Γ_2)-*cl*-atomic model of the theory T.

Definition 3. A set A_2 is called a fine almost weakly (Γ_1, Γ_2)-*cl*-algebraically prime in the theory T, if

1) A_2 is a fine almost weakly (Γ_1, Γ_2)-*cl*-atomic set of theory T;

2) $cl(A_2) = M_2, M_2 \in E_T \cap AP_T$, where AP_T is a class of algebraically prime models the theory T;

and obtained model M_2 is called a fine almost weakly (Γ_1, Γ_2)-*cl*-algebraically prime model of the theory T.

And in the frame above mentioned notions one of the obtained result is the following theorem:

Theorem 4. *Let T be complete for $\exists$-sentences perfect Jonsson theory and we have a fine almost weakly (Σ_1, Σ_1)-cl-atomic set of A_1 and a fine almost weakly (Σ_1, Σ_1)-cl algebraically prime set of A_2. Then $M_1 = cl(A_1)$ isomorphic to $M_2 = cl(A_2)$.*

All additional information regarding Jonsson theories can be found in [3].

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▸ AIBAT YESHKEYEV, OLGA ULBRIKHT, AND NAZERKE MUSSINA, *On the categoricity of the class of the Jonsson spectrum.*
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Let L be countable language of an arbitrary signature σ and $\mathcal{A}$ be an arbitrary model of this signature, i. e. $\mathcal{A} \in Mod\,\sigma$. Let us call *the Jonsson spectrum* of model $\mathcal{A}$ a set:

$$JSp(\mathcal{A}) = \{T \mid T \text{ is Jonsson theory in language } L \text{ and } \mathcal{A} \in Mod\,T\}.$$

The relation of cosemanticness on a set of theories is an equivalence relation. Then $JSp(\mathcal{A})/_{\bowtie}$ is the factor set of Jonsson spectrum of the model $\mathcal{A}$ with respect to $\bowtie$. [1]

Denote by $E_{[T]} = \bigcup\limits_{\nabla \in [T]} E_{\nabla}$ the class of all existentially closed models of class $[T] \in JSp(\mathcal{A})/_{\bowtie}$, where E_{∇} is a class of all existentially closed models of ∇.

A formula $\varphi(\bar{x})$ is called a Δ-formula [2] with respect to the theory T if there are existential formulas $\psi_1(\bar{x})$ and $\psi_2(\bar{x})$ such that $T \models (\varphi \leftrightarrow \psi_1)$ and $T \models (\neg\varphi \leftrightarrow \psi_2)$.

We say that a theory T admits R_1 [2], if for any existential formula $\varphi(\bar{x})$ consistent with T there is a formula $\psi(\bar{x}) \in \Delta$ consistent with T such that $T \models (\psi \to \varphi)$.

And in the frame above mentioned notions we have the following results.

Theorem 1. *Let $\mathcal{A}$ be an arbitrary model of signature σ, $[T] \in JSP(\mathcal{A})/_{\bowtie}$ and $[T]$ be complete for $\exists$-sentences class of universal theories for which holds R_1. Then the following are equivalent:*

1) the theory $[T]^*$ is ω_1-categorical,

2) any countable model from $E_{[T]}$ has an algebraically prime model extension in $E_{[T]}$.

Theorem 2. *Let L be a countable language, $\mathcal{A}$ be an arbitrary model of this language L, $[T] \in JSp(\mathcal{A})/_{\bowtie}$. If $[T]$ is $\forall\exists$-complete ω-categorical class, then $[T]$ has ω-categorical model companion $[T]^M$.*

Theorem 3. *Let L be a countable language, $\mathcal{A}$ be an arbitrary model of this language, $[T] \in JSp(\mathcal{A})/_{\bowtie}$. If $[T]$ is $\forall\exists$-complete $\varkappa$-categorical class, then $[T]^*$ is model complete.*

All additional information regarding Jonsson theories can be found in [3].

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[3] Yeshkeyev A.R., Kassymetova M.T., ***Jonsson theories and their classes of models***, Monograph, KSU, 2016.

► AIBAT YESHKEYEV AND OLGA ULBRIKHT, *The Jonsson nonforking notion under some positiveness.*
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Let L be a first-order language. Denote by At the set of atomic formulas of the language L and by $B^+(At)$ is the set of all positive Boolean combinations (conjunction and disjunction) of atomic formulas. $L^+ = Q(B^+(At))$ is a set of formulas in normal prenex form obtained by applying quantifiers ($\forall$ and $\exists$) to $B^+(At)$. A formula will be called positive if it belongs to L^+. Let Π_2^+ be the set of all $\forall\exists$-formulas of a language L^+. Let $\Delta \subseteq \Pi_2^+ \subseteq L^+$. All morphisms which we are considering below will be immersions as in [1].

Definition 1. Theory T will be called Δ-J-theory, if it satisfies the following conditions:
1) theory T has infinite model;
2) theory T is Π_2^+-axiomatizable;
3) theory T admits Δ-JEP;
4) theory T admits Δ-AP.

Let T be a Δ-J-theory, M is the semantic model of T. Let $\mathcal{A}$ be the class of all subsets of semantic model M and $\mathcal{P}$ is the class of all positive $\exists$-types (not necessarily complete), let $PJNF$ (positive Jonsson nonforking) $\subseteq \mathcal{P} \times \mathcal{A}$ be a binary relation. There is the list of the axioms 1-7 which defined positive Jonsson nonforking notion $PJNF$ and we have result for Δ-J-theory T.

Theorem 2. *The following conditions are equivalent:*
1) the relation $PJNF$ satisfies the axioms 1-7 relative to Δ-J-theory T;
2) T^* stable and for all $p \in \mathcal{P}$, $A \in \mathcal{A}$ ($(p, A) \in PJNF \Leftrightarrow p$ not fork over A) (in the classical meaning of S.Shelah), where T^* is the center of the Δ-J-theory T.

Further we considered on the $\mathcal{P} \times \mathcal{A}$ the relation $PJNFLP$ which is Δ-positive analog of the notion of forking by Lascar-Poizat [2]. The following theorem was obtained.

Theorem 3. *Let T be J-stable existentially complete perfect Δ-J-theory, then the following conditions are equivalent:*
1) the relation $PJNFLP$ satisfies axioms 1-7;
2) the concepts of $PJNF$ and $PJNFLP$ coincide.

All concepts that are not defined in this note will be able extract from [3].

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► PEDRO H. ZAMBRANO, DAVID REYES, *Co-quantale valued logics.*

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In this talk, we present a generalization of Continuous Logic (see [1]) where the distances take values in suitable co-quantales (in the way as it was proposed in [2]).

Co-quantales are somehow an interesting setting because R. Flagg ([2]) proved that any general topological space can be viewed as a generalized pseudo-metric space where the distance takes values on a suitable co-quantale.

By assuming suitable conditions (e.g., being co-divisible, co-Girard and a V-domain), we provide, as test questions, a proof of a version of the Tarski-Vaught test and Łoś Theorem in our setting.

Hopefully, this approach would provide an interesting setting to do Model Theory for general Topological Spaces.

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PPR Proofs and Programs

Invited talks

ORGANIZER:
Monika Seisenberger

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▸ HELMUT SCHWICHTENBERG, *Proofs and computation with infinite data.*
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It is natural to represent real numbers in [-1,1] by streams of signed digits -1,0,1. Algorithms operating on such streams can be extracted from formal proofs involving a unary coinductive predicate CoI on (standard) real numbers x: a realizer of CoI(x) is a stream representing x. We address the question how to obtain bounds for the lookahead of such algorithms: how far do we have to look into the input streams to compute the first n digits of the output stream? We present a proof-theoretic method how this can be done. The idea is to replace the coinductive predicate CoI(x) by an inductive predicate I(x,n) with the intended meaning that we know the first n digits of a stream representing x. Then from a formal proof of I(x,n+1) $\to$ I(y,n+1) $\to$ I(1/2(x+y),n) we can extract an algorithm for the average function whose lookahead is n+1 for both arguments. – This is joint work with Nils Koepp.

▸ THOMAS POWELL, *Some recent work in proof mining.*
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In this talk I will present some recent results on the application of proof theoretic methods in functional analysis, which focus on producing rates of convergence for algorithms that compute fixpoints for a specific class of contractive mappings. This talk will not assume any background in either proof theory or functional analysis but will instead aim to provide a high-level illustration of some of the core ideas that are relevant to applied proof theory in general.

► VINCENT RAHLI, *Brouwerian Intuitionistic Realizability Theories*.
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In this talk, I will present two Brouwerian intuitionistic realizability theories, namely BITT and OpenTT, whose underlying notions of computability go beyond that of standard Church-Turing. These two time-relative theories capture, through Brouwer's concept of choice sequences, the intuitionistic notion that new knowledge can be acquired as time progresses. We will describe how these two theories capture intuitionistic theories of choice sequences. In addition, we will discuss the status of the Law of Excluded Middle (LEM) w.r.t., these two theories. LEM, which essentially flattens the notion of time stating that it is possible to decide whether or not some knowledge will ever be acquired, can be shown to be false in BITT. It is however consistent with OpenTT, which relies on a more relaxed model of time, which is more classically inclined than BITT's.

As BITT and OpenTT are both inspired by CTT (a Brouwerian intuitionistic realizability theory implemented by the Nuprl proof assistant), we will start with a description of CTT. We will in particular describe how we were able to extend CTT with Brouwer's continuity principle for numbers as well as his bar induction principle (which allows deriving induction principles for W types), by validating these principles using our implementation of CTT in Coq.

This is joint work with Liron Cohen (Ben-Gurion University, Israel), and Mark Bickford Bob Constable (Cornell University, USA).

▸ DOMINIQUE LARCHEY-WENDLING (JOINT WORK WITH JF MONIN), *Extraction of recursive algorithms in Coq using the Braga method.*
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We present the Braga method which we use to get verified OCaml programs by extraction from fully specified Coq terms. Unlike structural recursion which is accepted as is by Coq, the Braga method works systematically with more involved recursive schemes, including the nonterminating schemes of partial algorithms, nested or mutually recursive schemes, etc. The method is based on two main concepts linked together: an inductive description of the computational graph of an algorithm and an inductive characterization of its domain. The computational graph mimics the structure of recursive calls of the algorithm and serves both (a) as a guideline for the definition of a domain predicate of which the inductive structure is compatible with recursive calls; and (b) as a conformity predicate to ensure that the Coq algorithm logically reflects the original algorithm at a low-level.

Contributed talks

► KATALIN BIMBÓ, *Abaci running backward.*
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Abacus machines were introduced by Lambek in [3]. (See [2] for a newer and easily accessible presentation.) Abacus machines are a full model of computation, which are equivalent to Turing machines, Markov algorithms, etc. They are ingenious in having only two kinds of instructions while also being deterministic (and computing functions on $\mathbb{N}$). A way in which a logic may be connected to a model of computation is through proofs, for example, proofs in a sequent calculus. In [1], we raised a problem for certain undecidability proofs by pointing out that the sequent calculus proofs, on which the undecidability claims rely, model *backward computation*.

In this talk, I define the notion of *reverse computation* for an abacus—following similar notions for finite state automata and finite state transducers introduced earlier. To ensure that reverse computation is as flexible as it reasonably can be, reverse computation is defined as a non-deterministic notion. Then, I prove that reverse computation in abaci is not sufficiently powerful to compute primitive recursive functions.

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▸ NICOLA BONATTI, *Two questions concerning quantifiers rules.*
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NK systems are distinguished on whether they adopt subordinate proofs for the rules of the quantifiers ∀ and ∃, thus distinguishing between indirect rules (Existential Elimination, Universal Introduction – see [1]) and direct rules (Existential Instantiation and Universal Generalisation – see [4]). Even if the rules are logically equivalent, the choice between direct and indirect rules has raised philosophical discussion on the role of eigenvariables in proofs. More precisely, as suggested by [2], the *descriptive* question concerning the role of eigenvariables in proofs should be distinguished from the *normative* question of what grounds the restrictions on both direct and indirect rules. In this talk, I will first argue that both direct and indirect quantifiers rules represent the same order relation – called term-dependence – among the eigenvariables introduced within a proof (either by direct or indirect rules). The order relation of term-dependence represents and constraints the choice process of instances for consecutive application of (in)direct rules – thus answering the *normative* question. Then, I will point out that term-dependence is instantiated in NK_{ε} (namely, NK extended with Hilbert's ε-operator – see [3]) at the syntactic level of nested ε-terms. I will conclude that term-dependence is best represented by NK_{ε}, where the ε-terms themself are interpreted as eigenvariables – thus answering the *descriptive* question.

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► YONG CHENG, *The interpretation degree structure of r.e. theories for which the first incompleteness theorem holds.*
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This work is motivated from finding the limit of the first incompleteness theorem ($\mathsf{G1}$) w.r.t. interpretation. We say that $\mathsf{G1}$ holds for a r.e. theory T iff for any recursively axiomatizable consistent theory S, if T is interpretable in S, then S is incomplete. It is natural to examine the interpretation degree structure and the Turing degree structure of r.e. theories for which $\mathsf{G1}$ holds.

Robinson's theory $\mathbf{R}$ is introduced in [3]. Given r.e. theories U and V, $U \trianglelefteq V$ means that U is interpretable in V, and $U \lhd V$ means that $U \trianglelefteq V$ but $V \trianglelefteq U$ does not hold; $U \leq_T V$ means that U is Turing reducible to V, $U <_T V$ means that $U \leq_T V$, but $V \leq_T U$ does not hold. Define $\mathsf{D} = \{S : S \lhd \mathbf{R}$ and $\mathsf{G1}$ holds for the r.e. theory $S\}$ and $\overline{\mathsf{D}} = \{S : S <_T \mathbf{R}$, and $\mathsf{G1}$ holds for the r.e. theory $S\}$.

We first show that the structure $\langle \overline{\mathsf{D}}, \leq_T \rangle$ is as complex as the Turing degree structure of r.e. sets. The interpretation degree structure of r.e. theories extending Robinson's arithmetic $\mathbf{PA}$ is well known. However, the interpretation degree structure of r.e. theories weaker than Robinson's theory $\mathbf{R}$ is much more complex. In this work, we try to answer the open questions about the structure of $\langle \mathsf{D}, \trianglelefteq \rangle$ in [1].

THEOREM 1 (Shoenfield, [2]). *Let A be recursively enumerable and not recursive. Then there is a consistent axiomatizable theory T having one non-logical symbol which is essentially undecidable and has the same Turing degree as A.*

Albert Visser improves Theorem 1, and shows that for any r.e. set A, there are disjoint r.e. sets B and C with $B, C \leq_T A$ such that for any r.e. D which separates B and C, we have $A \leq_T D$. We say a r.e. theory U is Turing persistent iff for any r.e. theory V, if $U \trianglelefteq V$, then $U \leq_T V$. From Visser's this result and Theorem 1, we can show that for any r.e. Turing degree $\mathbf{0} < d \leq \mathbf{0}'$, there exists a Turing persistent theory T_d with Turing degree d for which $\mathsf{G1}$ holds.

About the structure of $\langle \mathsf{D}, \trianglelefteq \rangle$, we have:

- For any r.e. Turing degree $\mathbf{0} < d < \mathbf{0}'$, there exists a Turing persistent theory T_d with Turing degree d such that $T_d \in \mathsf{D}$.
- There are countably many elements of D which are incomparable under $\trianglelefteq$.
- $\langle \mathsf{D}, \trianglelefteq \rangle$ has no minimal element if we restrict to finitely axiomatized theories.
- There is a descending chain of elements of D under $\lhd$ with countable length.
- If $\langle \mathsf{D}, \trianglelefteq \rangle$ has a minimal element, then it is also a minimum, and it is not Turing persistent.

We find that whether $\langle \mathsf{D}, \trianglelefteq \rangle$ has a minimal element (or $\langle \mathsf{D}, \trianglelefteq \rangle$ is well founded) depends on the signature of the language. If the signature of the language is infinite, then $\langle \mathsf{D}, \trianglelefteq \rangle$ has a minimal element. If the signature of the language is finite, this question is more difficult and is under examination.

Acknowledgement: I would like to thank Albert Visser for the share of his preliminary note on his improvement of Theorem 1, and email communications about this topic.

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▸ HORATIU CHEVAL, *General metatheorems in proof mining.*
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Proof mining [1] is a research program within applied proof theory having as its goal the extraction of information hidden in non-constructive proofs. The extracted content may take the form of quantitative results such as uniform effective bounds, or of the weakening of certain premises. A crucial advance came in 2005, when Kohlenbach [2] proved the first general metatheorems guaranteeing a priori, under certain conditions, that such results can be obtained. These metatheorems are each applicable in the context of a certain class of mathematical structures, Kohlenbach initially providing versions for inner product spaces, normed spaces, bounded metric spaces, W-hyperbolic spaces or CAT(0) spaces. Since 2005, metatheorems for other classes of structures in optimization and nonlinear analysis have been developed, for example for $\mathbb{R}$-trees and totally bounded metric spaces.

By identifying in the systems used in the results we enumerated the common properties involved in the proofs of the metatheorems, we introduce a generalization of them to a unified logical system of a more abstract class of structures satisfying these properties, containing the restrictions to systems without dependent choice of the aforementioned metatheorems as particular instances, with the goal of facilitating the introduction of metatheorems for structures not previously approached.

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▸ ANAHIT CHUBARYAN, ARSEN HAMBARDZUMYAN, *On non-monotonous properties of some propositional proof systems.*
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We investigate the relations between the proof lines of non-minimal tautologies and its minimal tautologies for some propositional systems of classical and nonclassical logics.

Definition 1. A tautology of some logic is called ***minimal*** if the replacement result of all occurrences for each of its non-elementary subformulas by some new variable is not a tautology of the same logic.

Definition 2. A minimal tautology φ of some logic is ***minimal of some formula*** ψ if φ is ψ, or φ is the replacement result of all occurrences of some non-elementary subformulas of ψ by some new variable. We denote by $M(\psi)$ the set of all minimal tautologies of the tautology ψ.

We denote by $\boldsymbol{t}^{\boldsymbol{\phi}}(\varphi)$ the minimal possible value of the number of proof steps for all proofs of the tautology φ in the system ϕ.

Definition 3. The proof system ϕ is called ***t-monotonous*** if for every tautology ψ there is a minimal tautology φ, such that $\varphi \in M(\psi)$ and $\boldsymbol{t}^{\boldsymbol{\phi}}(\psi) = \boldsymbol{t}^{\boldsymbol{\phi}}(\varphi)$.

Definition 4. The proof system ϕ is called ***t-strongly monotonous*** if for every tautology ψ there is no minimal tautology φ, such that $\varphi \in M(\psi)$ and $\boldsymbol{t}^{\boldsymbol{\phi}}(\varphi) > \boldsymbol{t}^{\boldsymbol{\phi}}(\psi)$.

Theorem. The Frege systems, the sequent systems with cut rule and the systems of natural deductions of classical, intuitionistic and Johansson's logics are not t-monotonous, and consequently, are not t-strongly monotonous.

Proof is give by showing that for these systems there are sequences of tautologies ψ_n, every one of which has unique minimal tautologies φ_n such that for each n the minimal proof lines of φ_n are an order more than the minimal proof lines of ψ_n.

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► ANAHIT CHUBARYAN, SARGIS HOVHANNISYAN, HAYK GASPARYAN, *Comparison of two propositional proof systems by lines and by sizes.*
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The two main proof complexity characteristics (*lines* and *sizes*) are compared for two classes of formulas in some "weak" propositional proof system, based on generalization of splitting method, and in one of "strong" systems - Frege systems.

For any proof system ϕ and tautology φ we denote by t_{φ}^{ϕ} (l_{φ}^{ϕ}) the minimal possible value of lines (sizes) for all ϕ-proofs of tautology φ.

We compare propositional proof system $\boldsymbol{GS}$, based on generalization of splitting method, which is defined in [1], and system $\boldsymbol{\mathcal{F}}$ – one of well-known Frege systems.

Our formulas are:

$$\boldsymbol{DNF_n} = \bigvee_{(\sigma_1,\dots,\sigma_n)\in E^n} \mathop{\&}_{i=1}^{n} p_i^{\sigma_i} \quad (n \geq 1)$$

and

$$\boldsymbol{TTM_{n,m}} = \bigvee_{(\sigma_1,\dots,\sigma_n)\in E^n} \mathop{\&}_{j=1}^{m} \bigvee_{i=1}^{n} p_{i_j}^{\sigma_i} \quad (n \geq 1,\ 1 \leq m \leq 2^n - 1)$$

Main results are the following:

$$t_{\boldsymbol{DNF_n}}^{GS} = \Theta(n),\ t_{\boldsymbol{DNF_n}}^{\mathcal{F}} = \Omega(2^n)$$

and

$$l_{\boldsymbol{DNF_n}}^{GS} = 0(n2^n),\ l_{\boldsymbol{DNF_n}}^{\mathcal{F}} = \Omega((2^n)^2).$$

Earlier it is proved in [1] that for sufficiently big n and $\forall i$ $(1 \leq i < [n \log_n 2])$ for formulas $\varphi_n^i = TTM_{n,n^i}$ we have $\log_2 t_{\varphi_n^i}^{GS} = \Omega(n^i)$ and $\log_2 l_{\varphi_n^i}^{GS} = \Omega(n^i)$, and it is proved in [2] that $t_{\boldsymbol{TTM_{n,2^n-1}}}^{\mathcal{F}}$ and $l_{\boldsymbol{TTM_{n,2^n-1}}}^{\mathcal{F}}$ are polynomial bounded.

Comparative analysis of above results shows that the first system is better by both complexity characteristics for the first of considered formula classes, just as the second system is better for the other classes.

[1] An. Chubaryan, Arm. Chubaryan, *Bounds of some proof complexity characteristics in the system of splitting generalization*, ***Otechestv. Nauka w epokhu izmenenij***, vol. 10 (2015), 2(7), 11–14, (in Russian).

[2] S. R. Aleksanyan and An. A. Chubaryan, *The polynomial bounds of proof complexity in Frege systems*, ***Siberian Mathematical Journal***, vol. 50 (2009), no. 2, pp. 243–249.

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In [2] (see also [1], [3]) we presented full proof of the equalities NP = coNP = PSPACE. These results have been obtained by the novel proof theoretic tree-to-dag compressing techniques adapted to Prawitz's [5] Natural Deduction (ND) for propositional minimal logic coupled with corresponding Hudelmaier's sequent calculus [4]. Recall that conventional interpretation of ND assumes that derivations are rooted trees whose nodes are labeled with formulas that are ordered according to the inference rules allowed; top formulas and the root formula are called assumptions and conclusion, respectively. Proofs are derivations whose all assumptions are discharged [5]. We use more lliberal interpretation that allows dag-like derivations whose nodes are ordered as DAGs (: directed acyclic graphs), not necessarily trees. Obviously dag-like derivations can be exponentially smaller than corresponding tree-like ones (but note that our dag-like proofs require a special notion of correctness). We elaborated a method of twofold horizontal compression of arbitrary "huge" polynomial-height (though possibly exponential-weight) tree-like proofs ∂ into equivalent "small" polynomial-weight dag-like proofs ∂_0 containing only different formulas at every horizontal level, whose correctness is verifiable in polynomial time by a deterministic TM. First part of compression [1] is defined by plain deterministic recursion on the height that provides us with "small" polynomial-weight dag-like proofs in a modified ND that allows multiple-premise inferences. In the second part [2] we apply nondeterministic recursion to eliminate multiple premises and eventually arrive at "small" dag-like proofs ∂_0 in basic ND, as desired. As an application [3] we consider simple directed graphs G and canonical "huge" tree-like exponential-weight(though polynomial-height) normal deductions (derivations) ∂ whose conclusions are valid iff G have no Hamiltonian cycles. By the horizontal compression we obtain equivalent "small" polynomial-weight dag-like proofs ∂_0 and observe that the correctness of ∂_0 is verifiable in polynomial time by a deterministic TM. Since Hamiltonian Graph Problem is coNP-complete, the existence of such polynomial-weight proofs ∂_0 proves NP = coNP [2], [3]. Now consider problem NP =? PSPACE. We know that the validity problem in propositional minimal logic is PSPACE-complete. Moreover, minimal tautologies are provable in Hudelmaier's cutfree sequent calculus by polynomial-height tree-like derivations ∂. Standard translation into ND in question yields corresponding "huge" tree-like proofs ∂' that can be horizontally compressed into desired "small" dag-like polynomial-weight proofs ∂_0 whose correctness is deterministically verifiable in polynomial time. This yields NP = PSPACE [2].

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[2] ——— *Proof Compression and NP Versus PSPACE II*, ***Bulletin of the Section of Logic***, vol. 49 (2020), no. 3, pp. 213–230.

[3] ——— *Proof Compression and NP Versus PSPACE II: Addendum*, ***Bulletin of the Section of Logic***, to appear.

[4] J. Hudelmaier, *An $O(n \log n)$-space decision procedure for intuitionistic propositional logic*, ***Journal of Logic and Computation***, vol. 3 (1993), pp. 1–13.

[5] D. Prawitz, ***Natural deduction: a proof-theoretical study***, Almqvist & Wiksell, 1965.

▸ RAHELEH JALALI, *On hard theorems for substructural logics.*
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Given a proof system, how can we specify the "hardness" of its theorems? One way to tackle this problem is taking the lengths of proofs as the corresponding hardness measure. Following this route, we call a theorem hard when even its shortest proof in the system is "long" in a certain formal sense. Finding hard theorems in proof systems for classical logic has been an open problem for a long time. However, in recent years as significant progress, many super-intuitionistic and modal logics have been shown to have hard theorems. In this talk, we will extend the aforementioned result to also cover a variety of weaker logics in the substructural realm. We show that there are theorems in the usual calculi for substructural logics that are even hard for the intuitionistic systems.

In technical terms, for any proof system **P** at least as strong as Full Lambek calculus, **FL**, and polynomially simulated by the extended Frege system for some infinite branching super-intuitionistic logic, we present an exponential lower bound on the proof lengths. More precisely, we will provide a sequence of **P**-provable formulas $\{A_n\}_{n=1}^{\infty}$ such that the length of the shortest **P**-proof for A_n is exponential in the length of A_n. The lower bound also extends to the number of proof-lines (proof-lengths) in any Frege system (extended Frege system) for a logic between FL and any infinite branching super-intuitionistic logic. We will also prove a similar result for the proof systems and logics extending Visser's basic propositional calculus **BPC** and its logic BPC, respectively. Finally, in the classical substructural setting, we will establish an exponential lower bound on the number of proof-lines in any proof system polynomially simulated by the cut-free version of $\mathbf{CFL_{ew}}$.

▸ MARCIN JUKIEWICZ, AND DOROTA LESZCZYŃSKA-JASION, *Genetic algorithms in proof-search tasks*[1].
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The aim of our work is an optimization of proof-search in a sequent system by a Genetic Algorithm (GA). In [1] we report on the satisfactory preliminary results: our GA provides derivation trees which are significantly shorter than trees built in a more standard manner. Moreover, a trend that shows up on the examined data is that the difference in the size of trees between the standard approach and GA grows exponentially with the size of tested formulas.

Our solution had one weakness – as the complexity of sequents increased, the effectiveness in finding the correct proof decreased. The problem lies in too extensive search space that made it difficult for GA to find the correct proof. Therefore we focused on improving our previous solution and our goal was to reduce computations performed by GA. In the previous work, some repetitive patterns could be seen in most of the outlined derivation trees. Since these elements are repeatable, it is possible to include them in the tree-building algorithm. Then GA should select these elements that cannot be built into the algorithm, because they strongly depend on a formula.

[1] M. Jukiewicz, D. Leszczyńska-Jasion, and A. Czyż, *Genetic algorithms in proof-search tasks*, (2021), submitted for publication.

[1] This work was supported financially by National Science Centre, Poland, grant no 2017/26/E/HS1/00127.

▸ PETER KOEPKE, *The Naproche natural language proof assistant.*
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The natural language proof assistant $\mathbb{N}$aproche (Natural Proof Checking) stems from two long-term efforts to narrow the gap between informal and formal mathematics: the System for Automated Deduction (SAD) project at Kiev and Paris [7], and the Naproche project at Bonn [2, 1, 3, 5]. Some texts which are formalized and proof-checked in $\mathbb{N}$aproche come close to ordinary mathematical writing, as indicated in the following excerpt from a formalization of König's Theorem in cardinal arithmetic [6]. The LaTeX dialect of the $\mathbb{N}$aproche input language ForTheL (Formula Theory Language) allows immediate mathematical typesetting of input files. The $\mathbb{N}$aproche system as well as the example formalization is included in the latest edition of the Isabelle prover platform [4]

Theorem 1. *Let κ, λ be sequences of cardinals on D. Assume that for every element i of D $\kappa_i < \lambda_i$. Then*

$$\sum_{i \in D} \kappa_i < \prod_{i \in D} \lambda_i.$$

Proof. Proof by contradiction. Assume the contrary. Then

$$\prod_{i \in D} \lambda_i \leq \sum_{i \in D} \kappa_i.$$

Take a function G such that $\dot{\bigcup}_{i \in D} \kappa_i$ is the domain of G and $\times_{i \in D} \lambda_i$ is the image of G. ... . Define

$$\Delta(i) = \{G((n,i))(i) \mid n \text{ is an element of } \kappa_i\} \text{ for } i \text{ in } D.$$

...
Contradiction. ⊣

[1] Marcos Cramer, ***Proof-checking mathematical texts in controlled natural language***, PhD thesis, University of Bonn, 2013.

[2] Marcos Cramer, Peter Koepke, Daniel Kühlwein, and Bernhard Schröder, *The Naproche system*, 2009.

[3] Steffen Frerix and Peter Koepke, *Automatic proof-checking of ordinary mathematical texts*, ***Proceedings of the Workshop Formal Mathematics for Mathematicians***, 2018.

[4] Isabelle contributors, ***The Isabelle2021 release***, February 2021.

[5] Peter Koepke, *Textbook mathematics in the Naproche-SAD system*, ***Joint proceedings of the FMM and LML Workshops***, 2019.

[6] Julius König, *Zum Kontinuumsproblem*, ***Mathematische Annalen***, vol. 60 (1905), pp. 177–180.

[7] Andrei Paskevich, ***Methodes de formalisation des connaissances et des raisonnements mathematiques: aspects appliques et theoriques.*** PhD thesis, Universite Paris 12, 2007.

▸ MATEUSZ ŁEŁYK, *On Σ_1-uniform reflection over uniform Tarski biconditionals*.
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A reflection principle is a way of expressing the soundness of a given theory in the language of this theory. In particular, the uniform reflection principle for a formalized arithmetical theory S consists of all sentences of the form

$$\forall x\big(\mathrm{Prov}_S(\phi(\dot{x})) \to \phi(x)\big),$$

where $\phi(x)$ is a formula in the language of S and $\mathrm{Prov}_S(x)$ is the canonical provability predicate for S. If we allow ϕ only from a certain class of formulae Γ, then the resulting principle is called Γ-uniform reflection for S.

In [1] it is shown that iterations of Σ_n-reflection principles over weak truth theories provide a convenient and uniform tool for performing the ordinal analysis of theories of predicative strength. Theories of truth in study consist uniquely of uniform Tarski biconditionals for some language $\mathcal{L}$ (call it $\mathrm{UTB}^-(\mathcal{L})$) i.e. sentences of the form

$$\forall x\big(T(\phi(\dot{x})) \equiv \phi(x)\big),$$

where T is a fresh predicate which does not belong to $\mathcal{L}$ and $\phi(x)$ is an $\mathcal{L}$-formula. One of the important issues is to understand relations between iterated uniform reflection over uniform Tarski biconditionals and more well-known hierarchies of compositional truth predicates.

In our talk we sketch a proof that Δ_0-induction for the compositional truth predicate suffices for proving Σ_1-uniform reflection over $\mathrm{UTB}^-(\mathcal{L}_{\mathrm{PA}})$, where $\mathcal{L}_{\mathrm{PA}}$ denotes the language of arithmetic. This answers an open problem posed in [1].

[1] Lev D. Beklemishev, Fedor Pakhomov, *Reflection algebras and conservation results for theories of iterated truth*, ***Preprint, arXiv:1908.10302, 2019***

▸ YUKIHIRO MASUOKA AND MAKOTO TATSUTA, *Counterexample to cut-elimination in cyclic proof system.*
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A *cyclic proof system* or a *circular proof system*, whose proof figures are finite trees with cycles, is an alternative proof system to the proof system with explicit induction. Brotherston defined the cyclic proof system $\mathtt{CLKID}^{\omega}$ for first-order logic with inductive definitions [1]. Conjecture 5.2.4. of [1] states the cut rule could not be eliminated in $\mathtt{CLKID}^{\omega}$. We show that the conjecture is correct by giving a counterexample. The counterexample is a sequent which states that an inductive predicate of the addition implies another inductive predicate of the addition. We give a $\mathtt{CLKID}^{\omega}$ proof of the sequent with the cut rule and show that there is no $\mathtt{CLKID}^{\omega}$ proof of the sequent without the cut rule.

[1] J. Brotherston, Sequent Calculus Proof Systems for Inductive Definitions, PhD thesis, University of Edinburgh, 2006.

▸ MATTIAS GRANBERG OLSSON AND GRAHAM LEIGH, *A proof of conservativity of $\widehat{\mathrm{ID}}^{\mathrm{i}}_1$ over Heyting arithmetic via truth.*

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We present work in progress on a novel proof of the conservativity of the intuitionistic fix-point theory $\widehat{\mathrm{ID}}^{\mathrm{i}}_1$ over Heyting arithmetic (HA), originally proved in full generality by Arai [1]. We make use of the work of van den Berg and van Slooten [2] on realizability in Heyting arithmetic over Beeson's logic of partial terms (HAP). The proof is divided into four parts: First we extend the inclusion of HA into HAP to $\widehat{\mathrm{ID}}^{\mathrm{i}}_1$ into a similar theory $\widehat{\mathrm{ID}}^{\mathrm{i}}_1\mathrm{P}$ in the logic of partial terms. We then show that every theorem of this theory provably has a realizer in the theory $\widehat{\mathrm{ID}}^{\mathrm{i}}_1\mathrm{P}(\Lambda)$ of fix-points for almost negative operator forms only. Constructing a hierarchy stratifying the class of almost negative formulae and partial truth predicates for this hierarchy, we use Gödel's diagonal lemma to show $\widehat{\mathrm{ID}}^{\mathrm{i}}_1\mathrm{P}(\Lambda)$ is interpretable in HAP. Finally we use the result of [2] that adding the schema of "self-realizability" for arithmetic formulae to HAP is conservative over HA.

[1] Toshiyasu Arai, *Quick Cut-elimination for Strictly Positive Cuts*, ***Annals of Pure and Applied Logic***, vol. 162 (2011), no. 10, pp. 807–815.

[2] Benno van den Berg and Lotte van Slooten, *Arithmetical Conservation Results*, ***Indagationes Mathematicae***, vol. 29 (2018), pp. 260–275.

▸ VALERIA DE PAIVA AND SAMUEL G. DA SILVA, *Dialectica and Kolmogorov-Veloso problems.*
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Blass' paper on questions and answers makes a surprising connection between Dialectica categories (models of Linear Logic), Vojtas' methods to prove inequalities between cardinal characteristics of the continuum (Set Theory) and complexity theoretical notions of problems (and reductions) between these. We recently realized that Kolmogorov's very abstract notion of problem, which is not related to specific complexity issues, can also be intrinsically related to Blass' examples above. Kolmogorov's notion of abstract problem, produces an alternative intuitive semantics for Propositional Intuitionistic Logic, an essential component of the celebrated Brouwer-Heyting-Kolmogorov (BHK) interpretation. We connect Kolmogorov's problems to objects of the Dialectica construction, thereby connecting also Veloso's problems. More importantly, we show how category theory gives us a better approach to Kolmogorov's problems, providing the morphisms that Kolmogorov lacked in 1932. Time allowing, we will discuss possible applications of these problems to multi-agent systems in Artificial Intelligence.

► LUIZ CARLOS PEREIRA and ELAINE PIMENTEL and VALERIA DE PAIVA, *Ecumenic Negation: one or two?*.
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Prawitz proposed an ecumenical system where classical logic and intuitionist logic co-exist harmoniously. Previous work on this Ecumenical Logic system has provided a Gentzen Natural Deduction formalization as well as a Gentzen sequent calculus formulation with the expected properties of normalization and cut-elimination. In these formulations Intuitionistic Propositional Logic and Classical Propositional logic, traditionally considered rival logics, accept and reject the same theorems. The ecumenical system as described has two disjunctions and two implications (one classical and one intuitionistic), but only one conjunction, one negation and one constant for falsum. Given that usually negation is defined as implication into falsum, it would seem reasonable to expect two negations, one the intuitionistic implication into falsum, the other a classical implication into falsum. However it is easy to prove that these two possible negations are interderivable in the Ecumenical system. Is this a sufficient criterium to decide on a single negation? This paper presents two arguments to defend the thesis that in fact there is only one way to assert the negation of a proposition A. The first argument is based on Glivenko's theorems and the second on the notion of 'computational isomorphism'. We discuss these arguments, as well as the failure of Joyal's collapse in minimal logic, as subsidies for a robust notion of ecumenical negation.

▶ YAROSLAV PETRUKHIN, *Cut-free proof systems for non-standard modal logics based on* **S5**.
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In this report, we present several modifications of Restall's [3] hypersequent calculus for **S5**. They formalize logics which are based on **S5**, but instead of necessity ($\Box$) and possibility ($\Diamond$) operators they have one or a few of the following ones: non-contingency ($\triangle$), contingency ($\triangledown$), essence ($\circ$) or accident ($\bullet$) modalities. They are defined as follows: $\triangle A = \Box A \vee \Box\neg A$, $\triangledown A = \Diamond A \wedge \Diamond\neg A$, $\circ A = A \to \Box A$, and $\bullet A = A \wedge \Diamond\neg A$. The formal study of $\triangle$ and $\triangledown$ is due to Montgomery and Routley [2], $\circ$ and $\bullet$ is due to Marcos [1]. However, among these logics only a non-contingency version of **S5** has already had a sequent calculus developed by Zolin [4], but it is not cut-free. We are going to fill this gap and suggest the following hypersequent rules:

$$(\triangle\Rightarrow)\ \frac{A,\Gamma\Rightarrow\Delta\mid H\qquad \Theta\Rightarrow\Lambda,A\mid G}{\triangle A\Rightarrow\ \mid\Gamma\Rightarrow\Delta\mid\Theta\Rightarrow\Lambda\mid H\mid G}\qquad (\Rightarrow\triangle)\ \frac{\Rightarrow A\mid A\Rightarrow\ \mid H}{\Rightarrow\ \triangle A\mid H}$$

$$(\triangledown\Rightarrow)\ \frac{\Rightarrow A\mid A\Rightarrow\ \mid H}{\triangledown A\Rightarrow\ \mid H}\qquad (\Rightarrow\triangledown)\ \frac{A,\Gamma\Rightarrow\Delta\mid H\qquad \Theta\Rightarrow\Lambda,A\mid G}{\Rightarrow\triangledown A\mid\Gamma\Rightarrow\Delta\mid\Theta\Rightarrow\Lambda\mid H\mid G}$$

$$(\circ\Rightarrow)\ \frac{A,\Gamma\Rightarrow\Delta\mid H\qquad \Theta\Rightarrow\Lambda,A\mid G}{\circ A,\Theta\Rightarrow\Lambda\mid\Gamma\Rightarrow\Delta\mid H\mid G}\qquad (\Rightarrow\circ)\ \frac{\Rightarrow A\mid A,\Gamma\Rightarrow\Delta\mid H}{\Gamma\Rightarrow\Delta,\circ A\mid H}$$

$$(\bullet\Rightarrow)\ \frac{\Rightarrow A\mid A,\Gamma\Rightarrow\Delta\mid H}{\bullet A,\Gamma\Rightarrow\Delta\mid H}\qquad (\Rightarrow\bullet)\ \frac{A,\Gamma\Rightarrow\Delta\mid H\qquad \Theta\Rightarrow\Lambda,A\mid G}{\Theta\Rightarrow\Lambda,\bullet A\mid\Gamma\Rightarrow\Delta\mid H\mid G}$$

One should replace the rules for $\Box$ in Restall's hypersequent calculus for **S5** with these ones in order to have a cut-free hypersequent calculus for a version of **S5** with non-standard modalities.

[1] Marcos J., *Logics of essence and accident*, ***Bulletin of The Section of Logic***, vol. 34 (2005), no. 1, pp. 43–56.

[2] Montgomery H., Routley R., *Contingency and noncontingency bases for normal modal logics*, ***Logique et Analyse***, vol. 9 (1966), no. 35–36, pp. 318–328.

[3] Restall G., *Proofnets for S5: Sequents and circuits for modal logic*, ***Logic Colloquium 2005*** (Costas Dimitracopoulos, Ludomir Newelski, Dag Normann and John R. Steel, editors), Cambridge University Press, 2007, pp. 151–172.

[4] Zolin E., *Sequential reflexive logics with noncontingency operator*, ***Mathematical Notes***, vol. 72 (2002), no. 5–6, pp. 784–798.

▸ IVO PEZLAR, *A note on paradoxical propositions from an inferential point of view.*
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In a recent paper by Tranchini [1] an introduction rule for the paradoxical proposition ρ^* that can be simultaneously proven and disproven is discussed. This rule is formalized in Martin-Löf's constructive type theory (CTT) and supplemented with an inferential explanation in the style of Brouwer-Heyting-Kolmogorov semantics. I will, however, argue that the provided formalization is problematic because what is paradoxical about ρ^* from the viewpoint of CTT is not its provability, but whether it is a proposition at all.

The main issue with ρ^* lies in the circular nature of its introduction rule, more specifically, there is a negative occurrence of ρ^* in its premise (i.e., ρ^* appears as an assumption). This clashes with the general justification scheme of CTT: formation rules, which tell us how to form new propositions, should be justified by the corresponding introduction rules, which tell us what these propositions mean, i.e., how to prove them. In the case of the proposition ρ^*, this justification requirement is, however, not met, since the introduction rule that should explain the meaning of ρ^* presupposes that we already understand it. Consequently, the formation rules cannot be understood as justified.

The other two variants of ρ^* considered by Tranchini are shown to have analogous issues. These variants are: 1) a paradoxical proposition ρ with a negative self-reference operator ! and its inverse ¡ and 2) semi-paradoxical propositions σ and τ whose paradoxical nature does not come from self-reference, or negative self-reference, but from their circular meaning dependencies.

[1] Luca Tranchini, *Proof, Meaning and Paradox: Some Remarks*, ***Topoi***, vol. 38 (2019), no. 3, pp. 591–603.

▶ ALEXEJ PYNKO, *Finite Hilbert-style calculi for disjunctive and implicative finitely-valued logics with equality determinant.*
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The main result of the work is the fact that any propositional finitely-valued logic (viz., logical matrix) $\mathcal{M}$ with connectives in a [finite] propositional language L and *equality determinant* $\Im(p) \subseteq \mathrm{Fm}_L$ (in the sense of [1]) as well as (possibly, secondary) disjunction|implication ($\vee | \supset$) is axiomatized by any finite Hilbert-style calculus for the ($\vee | \supset$)-fragment of the classical logic supplemented by [finitely many] rules|axioms to be [effectively] constructed in the following way. Given any fixed total ordering $\leqslant$ of the finite set $\Im$ and any *L-sequential $\Im$-table $\mathcal{T}$ of rank* $(0, 0)$ *for* $\mathcal{M}$ (in the sense of [1]) to be found [effectively] (cf. Theorem 1 therein), let $\mathcal{A}$ be the [finite] set constituted by:

1. those of the finitely many L-sequents, true in $\mathcal{M}$, with disjoint left and right sides without repetitions, constituted by elements of $\Im$ and ordered according to $\leqslant$, which are minimal under subsumption *partial* (because, for all formulas $\eta(p)$ and $\zeta(p)$, $\eta = p = \zeta$, whenever $\eta(\zeta) = p$) ordering between *such* sequents to be treated as disjuncts of the first-order signature $L \cup \{D\}$ with function symbols in L and the only relation unary one D;
2. for each $\iota \in \Im$ and every nullary $c \in L$, that (unique) of the sequents $\iota(c) \vdash$ or $\vdash \iota(c)$, which is true in $\mathcal{M}$;
3. for each $\iota \in \Im$ and every $F \in L$ distinct from ($\vee | \supset$) of arity $n > 0$ such that $F(p) \notin \Im$, whenever $n = 1$, all those sequents, which are resulted from sequents in $(\lambda/\rho)_{\mathcal{T}}(\iota(F))$ by adding the formula $\iota(F(p_1, \dots, p_n))$ to their right/left sides.

Then, we have the [finite] set $\mathcal{B} \triangleq \{((\phi_0 \vee q, \dots, \phi_{k-1} \vee q) \vdash (\psi_0, \dots, \psi_{m-1}, q))|(\varnothing \vdash (\psi_0/q, \phi_{k-1}, \dots, \phi_0/, \psi_{m-1} \supset q, \dots, \psi_0 \supset q)) \mid k \in \omega \ni m| = / \neq 1, \bar{\phi} \in \mathrm{Fm}_L^k, \bar{\psi} \in \mathrm{Fm}_L^m, (\bar{\phi} \vdash \bar{\psi}) \in \mathcal{A}\}$ of L-sequents with non-empty right sides|" and empty left ones". (Note that $q \notin (\{p\} \cup \{p_i\}_{0 \neq i \in \omega})$ is a variable occurring in no sequent in $\mathcal{A}$.) Finally, the supplementary rules|axioms are as follows: for each $(\bar{\phi} \vdash \bar{\psi}) \in \mathcal{B}$, where $\bar{\phi} \in \mathrm{Fm}_L^k$ and $\bar{\psi} \in \mathrm{Fm}_L^m$, while $k, m \in \omega$, whereas $m \neq 0| = k$, the L-rule|-axiom $\{\phi_0, \dots, \phi_{k-1}\} \to (\dots(\psi_0(\vee | \subset) \dots)(\vee | \subset)\psi_{m-1})$, respectively. In view of Examples 1, 2 and 3 of [1], this universal [effective] construction is well applicable (and has been successfully applied) to [both disjunctive|implicative fragments of the classical logic and] arbitrary|implicative four-valued expansions of Belnap's "useful" four-valued logic [by finitely many connectives] |"as well as to [both arbitrary Łukasiewicz' finitely-valued logics and] certain implicative paraconsistent three-valued logics [with finitely many connectives like HZ, providing its first *finite* Hilbert-style axiomatization]".

[1] A. P. Pynko, *Sequential calculi for many-valued logics with equality determinant*, ***Bulletin of the Section of Logic***, vol. 33 (2004), no. 1, pp. 23–32.

▸ MICHAŁ SOCHAŃSKI, DOROTA LESZCZYŃSKA-JASION, *On the representation of logical formulas as cographs*[1].
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In our talk, we propose a novel method of representing semantic information contained in formulas of propositional logic in the language of graph theory. The method starts with creation of a syntax tree of a formula, with every subformula in the tree labeled with α or β – depending on their type according to Smullyan's uniform notation – and with every leaf corresponding to an occurrence of a literal. Such labeled tree can be used to construct a graph G – further denoted as 'semantic graph' – where $V(G)$ is the set of leafs of the tree, and two vertices x_i, x_j in G are connected by an edge if the lowest common ancestor of x_i and x_j in the tree is a formula of type α. The resulting graph turns out to be a cograph and its properties can be used to analyse certain semantic properties of the formula. The most important property of semantic graphs is that every maximal clique in G corresponds to a set of literals L, such that any valuation that satisfies L, satisfies the formula. In addition to that it is known that every cograph is a permutation graph, which allows a representation of formulas – or the semantic dependencies between occurrences of literals in formulas – as a permutation. Many properties of cographs translate to properties of permutations; for example, maximal cliques in the cograph correspond to decreasing subsequences in the permutation. Both cographs and permutations allow the construction of efficient algorithms, which may make such representation of particular interest for computational logic.

[1] This work was supported financially by National Science Centre, Poland, grant no 2017/26/E/HS1/00127.

▸ MICHAŁ SOCHAŃSKI, DOROTA LESZCZYŃSKA-JASION, SZYMON CHLEBOWSKI, AGATA TOMCZYK, ALEKSANDER KIRYK, MARCIN JUKIEWICZ, *Synthetic tableaux: minimal tableau search heuristics*[1].
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In [1] we report on research on heuristics for generating minimal synthetic tableaux (ST) for CPL. The research was conducted in a quasi-experimental setting. Based on theoretical considerations we described a number of functions indicating heuristics of an optimal ST construction, and we developed a methodological framework to examine the efficiency of these functions.

Functions were tested on over 30 million of ST for more than 240 000 of formulas. The outcomes are satisfactory: we have settled the most efficient functions indicating heuristics to use them on larger data; also the methodological framework has been tested with a preliminary success.

In our talk we present the outcomes of further experiments conducted on data sets containing randomly generated formulas longer than those used in the first phase of research.

[1] M. Sochański, D. Leszczyńska-Jasion, Sz. Chlebowski, A. Tomczyk, M. Jukiewicz, *Synthetic tableaux: minimal tableau search heuristics*, (2021), submitted for publication.

[1] This work was supported financially by National Science Centre, Poland, grant no 2017/26/E/HS1/00127.

▶ AMIRHOSSEIN AKBAR TABATABAI, *Feasible Visser-Harrop property for intuitionistic modal logics.*
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A proof system P has feasible Visser-Harrop property, if there is a polynomial time algorithm that reads a P-proof of $\Gamma, \{A_i \to B_i\}_{i=1}^n \vdash A_{n+1} \vee A_{n+2}$ and produces a P-proof of $\Gamma, \{A_i \to B_i\}_{i=1}^n \vdash A_i$, for some $1 \leq i \leq n+2$, where Γ is a set of Harrop formulas.

In this talk, we will present a class of rules called the *almost positive rules* to show that any proof system for an intuitionistic modal logic that consists of theses rules, the cut and the necessitation rule has the feasible Visser-Harrop property. This method uniformly proves the property for the usual sequent-style and Hilbert-style proof systems for a broad range of intuitionistic modal logics, including IK, IKT, IK4, IS4, IS5, their Fisher-Servi versions, the intuitionistic logics for bounded depth and bounded width and the propositional lax logic. On the negative side, though, it shows that if an intuitionistic modal logic does not admit the Visser rules or specially does not have the disjunction property, then it does not have a calculus consisting only of almost positive rules, the cut rule and the necessitation rule. As the class of these rules is a general and natural class to consider, this negative result presents an interesting proof theoretical result about generic proof systems and their existence. This is based on a joint work with Raheleh Jalali.

▸ TOMCZYK AGATA*, GAWEK MARTA**, CHLEBOWSKI SZYMON*, *Natural Deduction Systems for Intuitionistic Logic with Identity.*
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The aim of our work is to present two Natural Deduction systems for Intuitionistic Sentential Calculus with Identity, (hereafter ISCI) [1]. The syntactically motivated Natural Deduction, closely follows axiom schemes in Hilbert-style system for ISCI. The other, semantically motivated, follows Gentzen's idea that each connective has an introduction and elimination rule. We focus on the normalization and subformula property of the aforementioned systems. We prove that in the case of ISCI normalization does not imply subformula property, however, a weaker version of subformula property still holds — the occurrences of non-subformulas in normal proofs can be constrained.

[1] Szymon Chlebowski, Dorota Leszczyńska-Jasion, *Investigation into Intuitionistic Logic with Identity*, ***Bulletin of the Section of Logic***, vol. 48 (2019), no. 4, pp. 259–283.

[2] Roman Suszko, *Abolition of the Fregean Axiom*, ***Lecture Notes in Mathematics***, vol. 453 (1975), pp. 169–239.

▶ BARTOSZ WCISŁO, *Disjunctive correctness and sequential induction.*
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This talk concerns axiomatic truth theories. They are formed by adding to a fixed base theory strong enough to handle syntax (in our case this will be Peano Arithmetic, PA) a unary predicate $T(x)$ with the intended reading "x is a Gödel code a true sentence" along with axioms postulating that the constructed predicate indeed behaves like the notion of truth. One of the basic sets of axioms considered in this context postulates that T satisfies Tarski's compositional clauses. E.g., a conjunction of two sentences is true iff one of the conjuncts is. This theory is called CT^- (Compositional Truth).

By a theorem of Kotlarski, Krajewski, and Lachlan, CT^- is conservative over PA. On the other hand, if we add to CT^- the full induction scheme for the formulae containing the truth predicate, the resulting theory can prove consistency of arithmetic and thus it is not conservative over its base theory. Tarski Boundary programme tries to establish what precise assumptions have to be made about the truth predicate in order to assure that a theory with that predicate is not conservative over its base theory. It turns out that a number of natural and seemingly unrelated principles are all equivalent to the Global Reflection Axiom which states that any sentence provable in Peano arithmetic is true in the sense of the predicate T. One of the most striking such equivalences is that Global Reflection is equivalent to Disjunctive Correctness which states that an arbitrary disjunction of a finite sequence of arithmetical sentences is true iff one of the disjuncts is. The analogue of this principle for a fixed finite number of disjuncts is a consequence of compositional axioms, but the quantified statement turns out to be much stronger.

In this talk, we will present a (relatively) direct proof that Disjunctive Correctness is equivalent to a certain weak form of induction for the truth predicate, thus obtaining a more straightforward argument that it is not conservative over PA. The introduced method will actually allow us to show that already one side of disjunctive correctness, "every true disjunction has a true disjunct," is equivalent to the Global Reflection.

This is a joint work with Cezary Cieśliński and Mateusz Łełyk.

STH Set theory

Invited talks

Organizers:
Joan Bagaria
Christina Brech

Invited speakers:
Omer Ben-Neria, Einstein Institute of Mathematics, Hebrew University of Jerusalem, Israel
Sandra Müller, Kurt Gödel Research Center - University of Vienna, Austria
Giorgio Venturi, State University of Campinas, Brazil
Trevor Wilson, University of Miami

▸ OMER BEN-NERIA AND DOMINIK ADOLF, *Tree-like scales and free subsets of set theoretic algebras.*
Hebrew University.
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In his PhD thesis, Luis Pereira isolated and developed several principles of singular cardinals that emerge from Shelah's PCF theory; principles which involve properties of scales, such as the inexistence of continuous Tree Like scales, and properties of internally approachable structures such as the Approachable Free Subset Property. In the talk, I will discuss these principles and their relations, and present new results from a joint work with Dominik Adolf concerning their consistency and consistency strength.

▸ SANDRA MÜLLER, *The strength of determinacy when all sets are universally Baire.*
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The large cardinal strength of the Axiom of Determinacy when enhanced with the hypothesis that all sets of reals are universally Baire is known to be much stronger than the Axiom of Determinacy itself. In fact, Sargsyan conjectured it to be as strong as the existence of a cardinal that is both a limit of Woodin cardinals and a limit of strong cardinals. Larson, Sargsyan and Wilson showed that this would be optimal via a generalization of Woodin's derived model construction. We will discuss a new translation procedure for hybrid mice extending work of Steel, Zhu and Sargsyan and use this to prove Sargsyan's conjecture.

▸ TREVOR M. WILSON, *Characterizing strong cardinals, virtually strong cardinals, and other large cardinals by Löwenheim–Skolem properties.*
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Let us say that a logic L has the *Löwenheim–Skolem* (LS) property at a cardinal κ if every sentence of L with a model M also has a model M_0 of cardinality less than κ, and has the *Löwenheim–Skolem–Tarski* (LST) property at κ if in addition we may take M_0 to be a substructure of M. Magidor [1] proved that the least cardinal at which second-order logic $L^2_{\omega\omega}$ has the LST property equals the least supercompact cardinal. By weakening the LST property to the LS property and strengthening $L^2_{\omega\omega}$ to various fragments of infinitary second-order logic $L^2_{\infty\infty}$, we obtain similar characterizations of various other large cardinals.

Letting $L^2_{\omega\omega}(\vee_\infty\forall_\infty)$ be the fragment of $L^2_{\infty\infty}$ obtained from atomic formulas and their negations by the operations of infinitary disjunction, finitary conjunction, infinitary universal quantification, and finitary existential quantification, we show that the least cardinal at which $L^2_{\omega\omega}(\vee_\infty\forall_\infty)$ has the LS property equals the least strong cardinal. We also show that the least cardinal at which $L^2_{\omega\omega}(\vee_\infty\forall_\infty)$ has the weak LS property, which is the special case of the LS property in which M has cardinality exactly κ, equals the least measurable cardinal.

Letting $L^2_{\kappa\omega}(\vee_\infty\forall_\infty)$ be as above but also allowing $<\kappa$-ary conjunctions, we show that any given cardinal κ is strong if and only if $L^2_{\kappa\omega}(\vee_\infty\forall_\infty)$ has the LS property at κ, and is measurable if and only if $L^2_{\kappa\omega}(\vee_\infty\forall_\infty)$ has the weak LS property at κ. We also obtain analogous results for $L^2_{\kappa\omega}(\vee_\infty)$, which allows only finitary quantification. Namely, we show that any given cardinal κ is *virtually strong* (a new large cardinal property weaker than remarkability) if and only if $L^2_{\kappa\omega}(\vee_\infty)$ has the LS property at κ, and is completely ineffable if and only if $L^2_{\kappa\omega}(\vee_\infty)$ has the weak LS property at κ.

[1] Magidor, M. On the role of supercompact and extendible cardinals in logic. *Israel J. Math.*, 10(2):147–157, 1971.

▸ GIORGIO VENTURI, *On non-classical models of ZFC*.
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In this talk we present some recent developments in the study of non-classical models of ZFC. We will show that there are algebras that are neither Boolean, nor Heyting, but that still give rise to models of ZFC. This result is obtained by using an algebra-valued construction similar to that of the Boolean-valued models. Specifically we will show the following theorem.

THEOREM 1. *There is an algebra $\mathbb{A}$, whose underlying logic is neither classical, nor intuitionistic such that $\mathbf{V}^{\mathbb{A}} \models$ ZFC. Moreover, there are formulas in the pure language of set theory such that $\mathbf{V}^{\mathbb{A}} \models \varphi \wedge \neg\varphi$.*

The above result is obtained by a suitable modification of the interpretation of equality and belongingness, which are classical equivalent to the standard ones, used in Boolean-valued constructions.

Towards the end of the talk we will present an application of these constructions, showing the independence of CH from non-classical set theories, together with a general preservation theorem of independence from the classical to the non-classical case.

(This is a joint work with Sourav Tarafder and Santiago Jockwich)

Contributed talks

▸ MATTEO DE CEGLIE, *The V-logic multiverse and MAXIMIZE.*
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I argue that classical set theory, $ZFC(+LCs)$, is *restrictive* compared to the V-logic multiverse (a novel set theoretic multiverse developed by the author and Claudio Ternullo). This multiverse is based upon Friedman's Hyperuniverse and Steel's set-generic multiverse: like the Hyperuniverse, it uses the infinitary V-logic as background logic (this logic admits formulas of length less than the first successor of the least inaccessible cardinal, but only a finite block of quantifiers in front of them) and admits all kinds of outer models of V (produced by set-generic, class-generic, hyperclass forcing). Like Steel's set-generic multiverse, it is recursively axiomatisable and is rooted on a ground universe that satisfies ZFC. For this proof, I compare $ZFC+LCs$ and the V-logic multiverse, characterised as $ZFC + LCs+$ the Multiverse Axiom Schema (this axiom tells us that if a sentence φ is consistence in V-logic then there is an actual outer model of V satisfying it), following Maddy's methodological principle MAXIMIZE (introduced in [3]). According to this principle, when comparing two foundational theories we should prefer the one that can prove more isomorphism types. I claim that the V-logic multiverse, as opposed to $ZFC + LCs$, does exactly that. This is because in the V-logic multiverse theory we can prove the existence of proper, uncountable, extensions of V, that we cannot have in $ZFC + LCs$ (see [2]). In turn, these extra objects means we can realise more isomorphism types that are not available in $ZFC + LCs$, since in the V-logic multiverse we can prove the existence of iterable class sharps and, more importantly, maps between them (see [1]). Moreover, when moving from $ZFC + LCs$ to the V-logic Multiverse we are not losing anything: ZFC, all the large cardinals, inner models and V are still there. On the other hand, when moving from the V-logic multiverse to $ZFC + LCs$ we lose the actual outer models of V, iterable class sharps and iterable class sharp generated models. Thus, this latter theory is restrictive compared to the V-logic multiverse theory.

[1] Carolin Antos, Neil Barton, Sy-David Friedman, *Universism and extensions of V*, ***Review of Symbolic Logic***, FirstView (forthcoming), pp. 1–43.

[2] Neil Barton, *Forcing and the Universe of Sets: Must we lose insight?*, ***Journal of Philosophical Logic***, vol. 49 (2020), no. 4, pp. 575–612.

[3] Penelope Maddy, ***Naturalism in Mathematics***, Oxford University Press, 1998.

▸ GABRIEL CIOBANU, *Various notions of infinity for finitely supported structures.*
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Finitely supported structures are related to permutation models of Zermelo-Fraenkel set theory with atoms. For such a structures we focus only on a finite subset (its 'finite support') which can characterize the entire structure. More exactly, they are sets equipped with actions of the group of all permutations of a fixed (infinite) set A of atoms satisfying a certain finite support requirement; this requirement states that any element of such a set is left unchanged under the effect of each permutation of A that fixes pointwise finitely many atoms.

There exist several notions of infinity for finitely supported structures: Tarski infinity, Dedekind infinity, Mostowski infinity, etc. These notions are defined and studied, and several relationships between them are given. There are emphasized the similarities and differences between these new definitions of infinity for finitely supported structures. By presenting examples of finitely supported sets that satisfy a certain forms of infinity, while they do not satisfy other forms of infinity, we show that these notions of infinity are pairwise non-equivalent.

Examples of some finitely supported sets satisfying various forms of infinity (Tarski I, Tarski III, Dedekind, Mostowski, Ascending, Tarski II and Non-amorphous infinity) are presented shortly in the table below, where $\mathbb{N}$ is the set of natural numbers, $\wp_{fin}(X)$ is the finite powerset of X, $\wp_{fs}(X)$ is the set of all finitely supported subsets of X, $T_{fin}(A)$ is the set of all finite and injective tuples of elements from A, and Y_{fs}^X is the set of all finitely supported functions from X to Y.

Set	TI i	TIII i	D i	M i	Asc i	TII i	N-am.
A	No	No	No	No	No	No	No
$A+A$	No	No	No	No	No	No	Yes
$A\times A$	No	No	No	No	No	No	Yes
$\wp_{fin}(A)$	No	No	No	No	Yes	Yes	Yes
$T_{fin}(A)$	No	No	No	No	Yes	Yes	Yes
$\wp_{fs}(A)$	No	No	No	No	Yes	Yes	Yes
$\wp_{fin}(\wp_{fs}(A))$	No	No	No	No	Yes	Yes	Yes
A_{fs}^A	No	No	No	No	Yes	Yes	Yes
$T_{fin}(A)_{fs}^A$	No	No	No	No	Yes	Yes	Yes
$\wp_{fs}(A)_{fs}^A$	No	No	No	No	Yes	Yes	Yes
$A\cup\mathbb{N}$	No	No	Yes	Yes	Yes	Yes	Yes
$A\times\mathbb{N}$	No	Yes	Yes	Yes	Yes	Yes	Yes
$\wp_{fs}(A\cup\mathbb{N})$	No	Yes	Yes	Yes	Yes	Yes	Yes
$\wp_{fs}(\wp_{fs}(A))$	?	Yes	Yes	Yes	Yes	Yes	Yes
$A_{fs}^{\mathbb{N}}$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbb{N}_{fs}^A$	Yes	Yes	Yes	Yes	Yes	Yes	Yes

More details are available in the recent book
Foundations of Finitely Supported Structures: a set theoretical viewpoint available at
URL Address: https://www.springer.com/gp/book/9783030529611..

▶ LUKE GARDINER, *Countable exponent partition relations on the real line*.
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For linear order types ξ, τ, and σ, the partition relation $\xi \to (\sigma)^\tau$ is the statement that whenever L is a linear order of order type ξ and $F : [L]^\tau \to 2$ is a colouring of the subsets of L of order type τ, there is a subset of L of order type σ which is homogeneous (monochromatic) for the colouring F. In the usual setting where ξ is an ordinal, relations where the exponent τ is infinite are inconsistent with the Axiom of Choice, by a theorem of Erdős and Rado [1], but it is consistent with ZF without Choice that such *infinite exponent partition relations* can hold.

We characterise the countable order types τ for which a relation of the form $\lambda \to (\sigma)^\tau$ is consistent with ZF, where λ is the order type of the real line $\mathbb{R}$, and we show that such relations hold iff every set of reals is completely Ramsey, a statement which is known to be consistent with $\mathsf{ZF} + \mathsf{DC}$ relative to an inaccessible cardinal (e.g. by a result of Mathias [2], it holds in Solovay's model).

[1] P. Erdős, R. Rado, *Combinatorial theorems on classifications of subsets of a given set*, ***Proceedings of the London Mathematical Society***, vol. s3-2 (1952), pp. 417–439.

[2] A. R. D. Mathias, *Happy families*, ***Annals of Mathematical Logic***, vol. 12 (1977), no. 1, pp. 59–111.

► MARTINA IANNELLA, *The complexity of convex bi-embeddability among countable linear orders.*
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Consider the set LO of countable linear orders and the following "convex embeddability" relation among them:

$$L \trianglelefteq_{LO} M \text{ iff } L \text{ is isomorphic to a convex set in } M.$$

One easily gets that $\trianglelefteq_{LO}$ is an analytic quasi-order on the Polish space LO. We first show that, in contrast to the usual embeddability between linear orders, the relation $\trianglelefteq_{LO}$ is combinatorially complicated: it is not a well quasi-order, indeed it has both infinite descending chains and antichains of size the continuum.

Denote by $\bowtie_{LO}$ the equivalence relation on LO induced by $\trianglelefteq_{LO}$.

THEOREM 1. (i) *The isomorphism relation $\cong_{LO}$ between linear orders is Borel reducible to $\bowtie_{LO}$. In particular, $\bowtie_{LO}$ is a proper analytic equivalence relation.*
(ii) *There is a Baire measurable reduction from $\bowtie_{LO}$ to $\cong_{LO}$.*
(iii) *If X is a turbulent Polish G-space, then the equivalence relation induced by the group G on X is not Borel reducible to $\bowtie_{LO}$.*

In particular, $\bowtie_{LO}$ is not complete for analytic equivalence relations.

Finally, we define the "(finite) piecewise convex embeddability" on LO, denoted by $\trianglelefteq_{LO}^{<\omega}$: given $L, L' \in LO$, we write $L \trianglelefteq_{LO}^{<\omega} L'$ if L is the sum of k disjoint convex subsets $L_i \subseteq L$, with $i = 0, ..., k < \omega$, such that each $L_i \trianglelefteq_{LO} L'$ via some map f_i, and the $f_i(L_i)$'s are pairwise disjoint in L' and ordered by $<_{L'}$. We consider its associated equivalence relation $\bowtie_{LO}^{<\omega}$, and show the following result.

THEOREM 2. $E_1 \leq_B \bowtie_{LO}^{<\omega}$.

As a corollary, we have that $\bowtie_{LO}^{<\omega}$ is not Baire reducible to any orbit equivalence relation, and by *(ii)* of Theorem 1 it does not reduce to $\bowtie_{\mathrm{LO}}$.

This is joint work with Vadim Kulikov, Alberto Marcone, and Luca Motto Ros.

▶ VLADIMIR KANOVEI, *On the 'Definability of definable' problem of Alfred Tarski.*
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Alfred Tarski defined [1] D_{pm} to be the set of all sets of type p, type-theoretically definable by parameterfree formulas of type $\leq m$, and asked whether it is true that $D_{1m} \in D_{2m}$ for $m \geq 1$. Tarski noted that the negative solution is consistent because the axiom of constructibility $V = L$ implies $D_{1m} \notin D_{2m}$ for all $m \geq 1$, and he left the consistency of the positive solution as a major open problem. This was solved in [2], where it is established that for any $m \geq 1$ there is a generic extension of L, the constructible universe, where it is true that $D_{1m} \in D_{2m}$.

THEOREM 1. *If $Y \subseteq \omega \setminus \{0\}$, $Y \in L$, then there is a generic extension of L in which $D_{1m} \in D_{2m}$ holds for all $m \in Y$ but fails for all $m \geq 1$, $m \notin Y$.*

It follows that Tarski's sentences $D_{1m} \in D_{2m}$ are not only consistent, but also independent of each other. This gives a full solution of the Tarski problem.

The other theorem concerns the sets $D_p = \bigcup_m D_{pm}$; thus D_p is the set of all sets of type p, type-theoretically definable by formulas of any type.

THEOREM 2. *There is a generic extension of L in which $D_1 = \mathcal{P}(\omega) \in D_2$.*

This result was announced by Harrington [3] but never published.

Our methods are based on almost-disjoint forcing of Jensen and Solovay.

[1] ALFRED TARSKI, *A problem concerning the notion of definability*, ***Journal of Symbolic Logic***, vol. 13 (1948), pp. 107–111.

[2] VLADIMIR KANOVEI AND VASSILY LYUBETSKY, *On the 'definability of definable' problem of Alfred Tarski*, ***Mathematics***, vol. 8 (2020), no. 12, Article No 2214.

[3] LEO HARRINGTON, ***The constructible reals can be anything***. Preprint dated May 1974, Available at `http://logic-library.berkeley.edu/catalog/detail/2135`.

► BORIŠA KUZELJEVIĆ, STEVO TODORČEVIĆ, *Cofinal types on ω_2*.
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We will present the preliminary analysis of the class $\mathcal{D}_{\aleph_2}$, the class of directed sets whose cofinality is $\aleph_2$. We compare orders in $\mathcal{D}_{\aleph_2}$ using the notion of Tukey reducibility $\leq_T$, and we isolate some simple cofinal types in this class. We will explain why all of the simple types are pairwise non-equivalent. Then we proceed to show for which pairs E_1, E_2 of these simple types there is no directed set D such that $E_1 <_T D <_T E_2$. We also show that for the remaining pairs of these simple types, if GCH holds and there is a non-reflecting stationary subset of $S^2_0 = \{\alpha < \omega_2 : cof(\alpha) = \omega\}$, then there is a directed set which is strictly between them in the Tukey ordering.

▸ PAUL BLAIN LEVY, *Broad Infinity and generation principles.*
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This work, presented in detail in [2], has three main contributions:

- To introduce an (arguably intuitive) set-theoretic axiom scheme, called Broad Infinity.
- To show it provides powerful generation principles for families, and (assuming AC) for sets and ordinals.
- To show it is equivalent (assuming AC) to the widely studied Ord-is-Mahlo scheme: every closed unbounded class of ordinals contains a regular ordinal [1, 3].

The new scheme is presented as follows. Let $\mathfrak{T}$ denote the universal class.

Firstly we want $\mathsf{Start} \in \mathfrak{T}$ and $\mathsf{Build} : \mathfrak{T}^3 \to \mathfrak{T}$ such that Build is injective and never yields Start. The following achieves this:

$$\mathsf{Start} \stackrel{\text{def}}{=} \emptyset$$
$$\mathsf{Build}(x, y, z) \stackrel{\text{def}}{=} \{\{x\}, \{x, \{\{y\}, \{y, z\}\}\}\}$$

A *signature* consists of a set I and an I-indexed family of sets $(K_i)_{i\in I}$. A *broad signature* is a class function from $\mathfrak{T}$ to the class of all signatures.

Given a broad signature G, a set X is said to be *G-inductive* when the following conditions hold.

- $\mathsf{Start} \in X$.
- For any $x \in X$ with $Gx = (K_i)_{i\in I}$, and any $i \in I$ and K_i-tuple $[a_k]_{k\in K_i}$ of elements of X, we have $\mathsf{Build}(x, i, [a_k]_{k\in K_i}) \in X$.

A *set of all G-broad numbers* is a minimal (and therefore least) G-inductive set. The axiom scheme of *Broad Infinity* states that, for every broad signature G, there is a set of all G-broad numbers. Equivalently: the class of all G-broad numbers (i.e. the least G-inductive class, which can be constructed) is a set. We may visualize a G-broad number as a well-founded three-dimensional tree.

Here is an attempt to articulate the intuitive justification for Broad Infinity. For any G-broad number of the form $\mathsf{Build}(x, i, [a_k]_{k\in K_i})$, the signature $Gx = (K_i)_{i\in I}$ is obtained from x, which "has already been constructed". This seems to provide a clearly specified construction process.

[1] A. Lévy, *Axiom schemata of strong infinity in axiomatic set theory*, ***Pacific Journal of Mathematics***, vol. 10 (1960), no. 1, pp. 223–238.

[2] P. B. Levy, *Broad Infinity and Generation Principles*, arXiv 2101.01698.

[3] H. Wang, *Large sets*, ***Logic, foundations of mathematics, and computability theory***, Springer, 1977, pp. 309–333.

▸ PHILIPP LÜCKE, *Huge reflection*.
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Results of Bagaria and his collaborators show that a great variety of large cardinal notions, ranging from weakly inaccessible cardinals to *Vopěnka's Principle*, can be characterized through principles of *Structural Reflection*. These principles generalize the *Downward Löwenheim–Skolem Theorem* to classes of models defined through external set-theoretic properties. In my talk, I want to present recent progress towards characterizing large cardinal notions beyond *Vopěnka's Principle* through natural structural reflection principles. I will introduce a simple reflection principle, called *Exact Structural Reflection*, and show that its validity implies the existence of various large cardinals in the region between almost huge cardinals and rank–into–rank embeddings.

This is joint work with Joan Bagaria (Barcelona).

▸ SANTIAGO JOCKWICH, *Algebra-valued models and Priest's logic of paradox*.
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This talk contributes to the study of models of non-classical set theories. We explore the possibility of constructing algebra-valued models of set theory based on Priest's logic of paradox (LP). We first outline the difficulties of this approach. In particular, we show that we can build an algebra-valued model based on LP, though, we obtain a model where Leibniz's law of indiscernibility of identicals fails and we loose several basic set-theoretic properties. On top of this we end up with a strange ontology. Then, secondly, we show that we can overcome this difficulties by modifying the interpretation map for $\in$ and $=$ in our algebra-valued model. Given the modified interpretation map we build a non-classical model of ZF which has as internal logic Priest's Logic of Paradox and validates Leibniz's law of indiscernibility of identicals. Even though it was already shown in [1] that set theories based on LP are compatible with ZFC, the validity of Leibniz's law of indiscernibility of identicals opens up the possibility of constructing equivalence classes and thus producing a quotient model based on LP with a rich ontology. We end by discussing the possibility of adding classes to the ontology of our algebra-valued model.

[1] G. Priest, *What if? The exploration of an idea*, ***Australasian Journal of Logic*** , vol.14 (2017), no.1.

▶ SOURAV TARAFDER AND GIORGIO VENTURI, *ZF between classicality and non-classicality.*

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Using a model $\mathbf{V}$ of Zarmelo-Fraenkel set theory (ZFC) and a complete Boolean algebra $\mathbb{B}$ one can construct Boolean-valued model $\mathbf{V}^{(\mathbb{B})}$ of ZFC. This is done by assigning to every set theoretic sentence an algebraic (truth) value by means of a map $[\![.]\!]$; a sentence φ is said to be *valid* in $\mathbf{V}^{(\mathbb{B})}$, denoted by $\mathbf{V}^{(\mathbb{B})} \models \varphi$, if $[\![\varphi]\!] = \mathbf{1}$, the *top element* of $\mathbb{B}$. This construction was generalised in [1] to get algebra-valued models $\mathbf{V}^{(\mathbb{A})}$ of classical and non-classical set theories, where $\mathbb{A}$ is a *reasonable implication algebra* (RIA).

If we now fix a model $\mathbf{V}$ of ZF and an algebra $\mathbb{A}$, but change the notion of validity as $\mathbf{V}^{(\mathbb{A})} \models \varphi$ iff $[\![\varphi]\!] \in D$, where $D \subseteq \mathbb{A}$ is called a *designated set*, then we get a more liberal interpretation of this method. The new class of algebras found in this way will be called *reasonable implication designated algebra* (RIDA), whose properties will depend on the interaction between the operations and the designated set. We will show how RIDA's offer a generalisation of RIA's. If $\mathbb{A}$ is a RIDA then $\mathbf{V}^{(\mathbb{A})} \models$ NFF-ZF, the *negation-free fragment of* ZF. Finally a property regarding complementation will be added to RIDA to have a new algebra, *reasonable implication complemented designated algebra* (RICDA). For a RICDA, $\mathbb{A}$ we will show $\mathbf{V}^{(\mathbb{A})} \models$ ZF.

Class many examples of RICDA, $\mathbb{A}$ will be provided so that the logic of the algebra is non-classical, but the logic of the set theory corresponding to $\mathbf{V}^{(\mathbb{A})}$ is classical.

[1] Löwe, B., & Tarafder, S., *Generalized algebra-valued models of set theory.*, ***Review of symbolic logic***, vol. 8 (2015), no. 1, pp. 192–205.

▶ CHRISTOPHER TURNER, *Forcing axioms and name principles.*
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Forcing axioms are a well-known formal expression of the concept "V contains $\mathbb{P}$-filters which are close to being generic", where $\mathbb{P}$ is some interesting forcing. They say "take κ many dense open sets of $\mathbb{P}$; then we can find a filter $g \in V$ which meets a lot of them" (where the value of κ and the interpretation of "a lot" depends on the forcing axiom). A classic example is Martin's Axiom $\mathrm{MA}(\omega_1)$, which talks about all c.c.c. forcings with $\kappa = \omega_1$ and "a lot" being interpreted as "all".

Here, we introduce another class of axioms which express this "close to generic" concept in a different way: *name principles.* These say: "Let σ be any sufficiently nice name such that $\mathbb{P} \vdash P$. Then there is a filter $g \in V$ such that P is true for σ^g in V." Here, P is some first order property, which depends on the name principle. These name principles have been used on an ad-hoc basis, but have not been studied much in their own right. We present a small selection of the many connections between name principles and forcing axioms.

This is joint work with Philipp Schlicht.