# Lattice Structure of Some Closed Classes for Non-binary Logic and Its Applications

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**Abstract** The paper provides a brief overview of modern applications of multi-valued logic models, where the design of heterogeneous computing systems with small computing units based on three-valued logic gives the mathematically better and more effective solution compared to binary models. It is necessary for applications to implement circuits comprised from chipsets, the operation of which is based on three-valued logic. To be able to implement such schemes, a fundamentally important theoretical problem must be solved: the problem of completeness of classes of functions of three-valued logic. From a practical point of view, the completeness of the classes of such functions ensures that circuits with the desired operations can be produced from on an arbitrary (finite) set of chipsets. In this paper, the closure operator on the set of functions of three-valued logic, that strengthens the usual substitution operator has been considered. It was shown that it is possible to recover the sublattice of closed classes in the general case of closure of functions with respect to the classical superposition operator. The problem of the lattice of closed classes for the class of functions $T_2$ preserving two is considered. The closure operator $\mathcal{R}_1$ for which functions that differ only by dummy variables are considered to be equivalent is considered in this paper. A lattice is constructed for closed subclasses in $T_2 = \{f \,|\, f(2, \ldots, 2) = 2\}$ – class of functions preserving two

**Keywords** Three-valued logic application · Three-valued logic · Closure operator · Lattice structure · Closed subclasses · Substitution operator

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# 1 Introduction

A ternary system is the most optimal from the point of view of information density [9]. The generalization for multi-valued logic is the ternary logic [2, 3]. Further, without loss of generality instead of multivalued case a ternary logic model may be considered. In ternary logic, a statement is assigned one of three values: “true”, “false”, “undefined” [2, 4, 9]; in binary logic—two: either “true” or “false”. Symmetric form of number representation based on three-valued logic simplifies a processing of negative numbers, since it requires an extra bit to store the sign [4].

Some features of the operation logic of a ternary computer, for example, the representation of negative numbers, give possibilities for design more reliable and high-performance modern systems, that will be useful for many modern applications. Mathematically, ternary logic is more efficient than binary logic [2, 4, 9]. Research and development of algorithms based on three-valued logic are very relevant [8], for example, in telecommunications [7, 10], in the field of artificial intelligence (AI) [6], quantum computing [7, 11–13], medicine, physics [14]. This is confirmed by a significant increase of the number of scientific publications in leading scientific journals related to various applications of three-valued logic over the past few years [17].

## *1.1 A Brief Overview of Modern Applications of Multivalued Logic*

Here are examples of several applications where the construction of algorithms based on three-valued logic provides greater efficiency and turns out to be preferable in comparison with two-valued logic. For more detailed overview, you can read references.

**Reliability analysis of structural processes and factors assessment of technical systems** Multi-valued logic allows to consider qualitative variables instead of quantitative ones. Quantitative indicators (factors) are discretized by mapping into a certain $m$-interval scale. This approach allows you to combine quantitative and qualitative indicators within the single model. The reliability of the factors decreases minimally with such discretisation. This allows to investigate the model as fully as possible. This is especially effective in situations where there is no way to quantify the impact of a particular factor on the process. The use of qualitative variables provides additional opportunities for assessing factors.

**Simulation of processes and modern design languages** Simulation is the only available way to check the quality and reliability of complicated and expensive technical systems at their design stage. Automated design tools allow you to assess quality based on real-world operating conditions. Temporary simulation of circuits in an automated simulation system is often based on the principles of three-valued logic.

**Design of data transmission and processing systems** Ternary logic is effective in constructing computing units for equipment of data transmission networks. Potentially, the transmission of three states instead of two bits at a time can increase the data transfer rate by 1.5 times. With an increase of the number of trits (instead of bit) the speed can grow exponentially [10, 18, 19]. It is possible to implement solutions for data aggregation and transmission based on multivalued logic. These solutions will provide a single high-dimensional space for network addressing—both for standard purposes of data transmission [15] and for new tasks for controlling robotic devices for the Internet Of Things [7].

Three-valued logic is also effective both for solving problems of image processing [5] and for problems of cryptography. Quantum computing for data security is the most effective method of protecting mobile robots, the Internet of Things (IoT) and security of distributed applications. That also uses multi-valued logic models. With the rapid growth of quantum computers, ternary computing has become relevant again [5, 12, 13]. The leading IT companies have introduced their quantum computers operating on several dozen of qubits in the last decade: IBM quantum processors consist of 65 qubits, Google has 72 [20]. The developers plan to release a 1112-qubit processor called "Condor" by 2023, that should bring quantum technologies to a new commercial level [20].

Also, at present, the multi-valued logic toolkit is widely used in tasks related to data analysis and the construction of AI models, for example, in the tasks of hierarchical data clustering for arbitrary complicated data sets [6, 7]. Interpretation models via 3-valued logic allows to overcome exiting limitations on the ability to create fully automatic program-analysis algorithms [1].

At the end of this short overview of multivalued logic models, the application in economic research should be mentioned: models of collective behavior and the problem of collective choice, where "cyclical logic" arises as a special case of $k$-valued logic [21].

## 2 Theoretical Aspects of Designing of Computing Systems Based on Three-Valued Logic

All applied problems considered above are reduced to the problem of determining the factors that have an influence on the process and considering a countable set $P_3$ of states of these factors. Any countable number of states can be approximated by basically three states [23]: 0, 1, 2.

And for a decision making someone need to find the value of the output function $Y$ that depends on this set. Accordingly, the output function $Y$ can be represented as a combination of predicates on the set $P_3$ [22]. For this purpose complicated predicates and superpositions of these predicates on $P_3$ will be considered.

These predicates can be implemented (from practical point of view) as circuits of chips, the operation of which is based on three-valued logic.



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## 2.1 Completeness of Functions Classes of Three-Valued Logic

A fundamentally important problem—**the problem of completeness of classes of functions of three-valued logic** [22]—must be solved to make this implementation possible. From the practical point of view, the completeness of the classes of functions guarantees that a circuit with the desired functional diagram can be produced based on an arbitrary finite number of chipsets. For two-valued logic, this problem was also solved by Emil Post, which led to the explosive growth of electronics [24].

Post's classical theorem describes five precomplete classes in the set of Boolean functions [24].

For the case of three-valued logic, the problem was solved by Yablonsky in 1958 [22, 23]. He proved that there are 18 precomplete classes for functions of three-valued logic. In the papers [22, 23], the closure of the set of functions with respect to the substitution operator was considered.

Unfortunately, for three-valued logic it was proved that this problem cannot be solved in a general case [23]. If the lattice of closed classes is countable in the case of two-valued logic, then it is exponential in the case of three-valued logic. However, its closure operators on the set of three-valued logic functions can be considered, which are a strength of the common substitution operator.

Solving the completeness problems for this new closure operator and finding the structure of the lattice of closed classes will help not only to restore the sublattice of closed classes in the general case of closure of functions with respect to the classical superposition operator, but also will optimize the possible production of chips for functional circuits for solving the problem described above in the Introduction.

Consider a variant of the closure operator $\mathcal{R}_\infty$, for which functions that differ only in dummy variables are considered equivalent. Let us construct a lattice for closed subclasses in $T_1 = \{f \mid f(1, \ldots, 1) = 1\}$ — in the class of functions preserving two.



## 2.2 Lattice of Closed Subclasses $T_2$ with Respect to $\mathcal{R}_\infty$

**Definition 1** Let $f(x_1, \ldots, x_i, \ldots, x_n) \in P_3$, $|X_f| = n$, then $x_i$ called $\mathcal{R}_\infty$-essential for $f$, if there are sets $\alpha_1^n = (a_1, \ldots, a_{i-1}, b^1, a_{i+1}, \ldots, a_n)$, $\alpha_2^n = (a_1, \ldots, a_{i-1}, b^2, a_{i+1}, \ldots, a_n)$ such that $f(\alpha_1^n) \sim f(\alpha_2^n)$.

**Completeness in $T_2$**

**Definition 2** Use the following notation $T^{02} \stackrel{\text{def}}{=} \{f \mid \exists i \in \{1, X_f\} : \alpha = (a_1, \ldots, a_{X_f}), a_i \in \{0, 2\} \Rightarrow f(\alpha) = 2\}$

$T^{12} \stackrel{\text{def}}{=} \{f \mid \exists i \in \{1, X_f\} : \alpha = (a_1, \ldots, a_{X_f}), a_i \in \{1, 2\} \Rightarrow f(\alpha) = 2\}$

$T^{02} \stackrel{\text{def}}{=} \{f \mid \alpha = (a_1, \ldots, a_{X_f}); a_i \in \{0, 2\}, i \in \{1, X_f\} \Rightarrow f(\alpha) = 2\}$

$T^{12} \stackrel{\text{def}}{=} \{f \mid \alpha = (a_1, \ldots, a_{X_f}); a_i \in \{1, 2\}, i \in \{1, X_f\} \Rightarrow f(\alpha) = 2\}$

**Lemma 1** *The class $T^{02}-$ is $\mathcal{R}_\infty$-closed.*

**Lemma 2** *The class $T^{12}-$ $\mathcal{R}_\infty$ is closed.*

Proof of Lemma 1. Note that neither the permutation of variables nor identification or addition of inessential (dummy) ones affect the property functions belong to class $T^{02}$. This follows obviously from the class definitions.

It is also obvious that if $f \in T^{02}$, then for any function $g(f \sim g)$ it's true that $g \in T^{02}$.

Now show that the superposition of functions from the class $T^{02}$ will also lie in class $T^{02}$.

Let $f \in T^{02}$, $f = f(x_1, \ldots, x_n)$. Consider the function $h = f(g_1, \ldots, g_n)$, where $g_i-$ are either free variables or functions from the set $T^{02}$.

By contradiction, let $h \notin T^{02}$, then there is a set $\alpha = (a_1, \ldots, a_{|X_h|}), a_i \in \{0, 2\}, 1 \leq i \leq |X_h|$, such that it's true that $h(\alpha) \neq 2$.

And by the construction of the function $h$, and under the condition that $f \in T^{02}$ there is such $i$ that the function $g_i(\beta) \neq$ , where $\beta = (b_1, \ldots, b_{|X_{g_i}|}), 1 \leq b_i \leq |X_{g_i}|$—projection of vector $\alpha$ on the coordinate axes corresponding to free variables of the function $g_i$.

Thus the function $g_i \notin T^{02}$, but that contradicts the choice of function $g_i$. Thus $h \in T^{02}$.

The lemma 1 is proved.

The Lemma 2 can be proved by repeating the sketch of the proof of lemma 1 (by formal replacing of $T^{02}$ by $T^{12}$).

**Lemma 3** *The class $T^{02}-$ $\mathcal{R}_\infty$ is pre-complete in the class $T_2$.*

***Proof*** Note that the class $T_2 = \mathcal{R}_\infty(\{, \})$, where $f(|X_f| = 2)\&(f(\alpha) = 2$ if and only if when $\alpha = (2, 2))$, $g(|X_g| = 1)\&(g \in T_2)\&(g \notin T_\sim)$.

Let there be a function $w(w \notin T^{02})$. Then by definition there is a set $\alpha = (a_1, \ldots, a_{|X_w|}), a_i \in \{0, 2\}, 1 \leq i \leq |X_w|$ such that $w(\alpha) \neq 2$.

Let's move on from the function $w$ to function $w'$, derived from $w$ by identifying variables according to the set $\alpha$. Namely, variables in the set $\alpha$ will be identified with the same values. Thus, the whole set of variables of the function $w$ may be split into two groups: with respect to 0 and with respect to 2. By identification, that gives the function $w'(|X_{w'}| = 2)\&(w' \notin T^{02})$.

Let without loss of generality $w'(0, 2) = 1$. If this is not true, then by rearranging the variables and moving to function $w''(w'' \sim w')$ the function with the specified property can be obtained easily.

If the vector $\alpha$ does not contain elements equal to 2, then the function that $\sim$ a function $w'$ and satisfies the required properties may be considered.

Note that a function $g(g \in T^{02})\&(|X_g| = 1)\&(g \notin T_\sim)$ exists. Consider a function $w''(w'' \sim w)$ such that:

$$w''(\alpha) = \begin{cases} 1, \ \alpha = (2, 0) \\ 2, \ w'(\alpha) = 2 \\ 0, \ \text{otherwise.} \end{cases}$$



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Consider a function $v_1(x, y) = g(w''(x, y))$. The property $v_1(\alpha) = 1$ for this function holds if and only if when $\alpha = (0, 2)$. Also consider a function $v_2 = v_2(x, y) = v_1(y, x)$. It is easy to see that by construction it gives $\{v_1, v_2\} \subseteq \mathcal{R}_\infty(\mathcal{T}'^{\in} \cap \sqsupseteq)$.

Consider the function $d$ such that:

$$d(\alpha) = \begin{cases} 2, \ a_i \in \{0, 2\}, 1 \le i \le 2 \\ 1, \qquad\qquad \text{otherwise.} \end{cases}, \alpha = (a_1, a_2).$$

It's obviously that $d \in T^{02}$. Let's construct a function $m$:

$$m(x, y) = d(d(v_1(x, y), d(x, y)), v_2(x, y))$$

$$m(\alpha) = \begin{cases} 2, \ a_1 = 1, 1 \le i \le 2 \\ 1, \qquad\qquad \text{otherwise.} \end{cases}, \alpha = (a_1, a_2).$$

By the fact that the function $2 \in T^{02}$ a function $f$ can be constructed such that:

$$f(x, y) = m(m(x, 2), m(y, 2))$$

$$f(\alpha) = \begin{cases} 2, \ a_i = 2, 1 \le i \le 2 \\ 1, \qquad\qquad \text{otherwise.} \end{cases}, \alpha = (a_1, a_2).$$

It was mentioned above that $\mathcal{R}_\infty(\{\{,\}\}) = \mathcal{T}_\in$. But by construction it can be obtained that $f \in \mathcal{R}_\infty(\Updownarrow) \subseteq \mathcal{R}_\infty(\sqsupseteq, \mathcal{T}'^{\in})$, and by definition $g \in T^{02}$, therefore $T_2 = \mathcal{R}_\infty(\sqsupseteq, \mathcal{T}'^{\in})$.

The lemma is proved.

**Lemma 4** *Let $f \in T_2$ and $f \notin T_{01}$. Then $2 \in \mathcal{R}_\infty(\{)$*

***Proof*** Consider the function $h(x) = f(x, \dots, x)$. It is easy to show that if $h \notin T_{01}$, then $2 \in \mathcal{R}_\infty(\langle)$.

Let $h \in T_{01}$. Note that for any $g(|X_g| = 1)\&(g \in T_{01})$ it holds that $g \in \mathcal{R}_\infty(\langle)$. by condition $f \notin T_{01}$, hence there is a set $\alpha = (\alpha_1, \dots, \alpha_n), n = |X_f|, \alpha_i \in \{0, 1\}, 1 \le i \le n$ such that $f(\alpha) = 2$. Construct a function $f' = f(g_1, \dots, g_n), |X_{g_i}| = 1, g_i \in T_{01}, 1 \le i \le n$, at that $g_i(0) = \alpha_i$. Note that $\{g_i, h\} \subset \mathcal{R}_\infty(\{)$ therefore $f' \in \mathcal{R}_\infty(\{)$. Consider a function $h'(x) = f'(x, \dots, x)$. By construction it can be obtained that $h'(0) = h'(2) = 2$, and therefore according to the already considered case we have $2 \in \mathcal{R}_\infty(\langle') \subset \mathcal{R}_\infty(\{)$.

The lemma is proved.

**Lemma 5** *A class $T_\sim \cap T_2-$ is $\mathcal{R}_\infty-$precomplete in $T_2$.*

***Proof*** Let $f \notin T_\sim, f \in T_2, |X_f| = n$. Let us show that $\mathcal{R}_\infty(\{\{ \cup \mathcal{T}_\sim \cap \mathcal{T}_\in\}) = \mathcal{T}_\in$. Note that, by definition, there are at least two sets $\alpha_1 = (a_1^1, \dots, a_n^1)$ and $\alpha_2 =$

$(a_1^2, \ldots, a_n^2)$ such that $\alpha_1 \sim \alpha_2$, and $f(\alpha_1) \sim f(\alpha_2)$. Identify variables in $f$ according to the coincidence of identical pairs in vectors $\alpha_1$ and $\alpha_2$. Concretely if $(a_i^1, a_i^2) = (a_j^1, a_j^2)$, then $i-$th and $j-$th variables are identified. Thus the function $f'$ of five variables satisfying the following condition has been obtain

$$f'(0, 1, 2, 0, 1) \sim f'(0, 1, 2, 1, 0)$$

Without loss of generality, it can be assumed that after identification variables the function $f'$ will have exactly this order variables. Otherwise, the variables will be reordered. Also note that some of the variables of the function $f'$ can be dummy.

Note that there is $2, g \in T_{\sim} \cap T_2(g(0) = 1, g(1) = 0)$. Let's move on from the function $f'$ to a function $f'' = f'(g(x_1), x_1, 2, x_2, x_3)$, $f'' \in \mathcal{R}_\infty(\mathcal{T}_{\sim} \cap \mathcal{T}_{\in})$. A function $f''$ satisfies the property

$$f''(0, 0, 1) \sim f''(1, 0, 1)$$

Let without loss of generality $f''(1, 0, 1) = 2$.

There are functions $f \in T_{\sim} \cap T_2$, such that $f(\alpha) = 2$ if $\alpha = (2, \ldots, 2)$. Denote the set of such functions as $N$. Let us show by a construction that $\mathcal{R}_\infty(\{\{'',\mathrm{N}\}) = T_2$.

Let $h \in T_2$ – arbitrary function. Consider the functions $g_0, g_1, g_2 \in N(|X_{g_i}| = |X_h| = n)$.

$$g_0(\alpha) = \begin{cases} 2, \ \alpha = (2, \ldots, 2) \\ 0, \qquad \text{otherwise.} \end{cases}$$

$$g_1(\alpha) = \begin{cases} 2, \ \alpha = (2, \ldots, 2) \\ 1, \qquad \text{otherwise.} \end{cases}$$

$$g_2(\alpha) = \begin{cases} 2, \ \alpha = (2, \ldots, 2) \\ 1, \qquad h(\alpha) = 2, \\ 0, \qquad h(\alpha) \neq 2, \end{cases}$$

Consider the function $h'(x_1, \ldots, x_n) = f''(g_2(x_1, \ldots, x_n), g_1(x_1, \ldots, x_n), g_0(x_1, \ldots, x_n))$. By construction $h' \sim h$. Thereby $\mathcal{R}_\infty(\langle') = \mathcal{R}_\infty(\langle) \subseteq \mathcal{R}_\infty(\{\{'', \mathcal{N}\}) \subseteq \mathcal{R}_\infty(\{, \mathcal{T}_{\sim} \cap \mathcal{T}_{\in})$. Due to the arbitrariness of the function $h \in T_2$ we get $T_2 \in \mathcal{R}_\infty(\{\{, \mathcal{T}_{\sim} \cap \mathcal{T}_{\in}\})$.

The lemma is proved.

**Lemma 6** *A class $T_{01} \cap T_2$ – $\mathcal{R}_\infty$-precomplete in $T_2$.*

***Proof*** Consider the function $f \notin T_{01} \cap T_2$, $f \in T_2$. By Lemma 4 $\mathcal{R}_\infty(\in, \mathcal{T}_{\prime\infty} \cap \mathcal{T}_{\in}) \subseteq \mathcal{R}_\infty(\{, \mathcal{T}_{\prime\infty} \cap \mathcal{T}_{\in})$. Let $h \in T_2$ — arbitrary function from $T_2$. Note that there is a function $g \in T_{01} \cap T_2$, satisfying the following property:

$$g(0, 2) \sim g(1, 2)$$



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Without loss of generality $g(1, 2) = 2$.

Consider the function $m \in T_{01} \cap T_2(|X_m| = |X_h| = n)$ such that:

$$m(\alpha) = \begin{cases} 2, & \alpha = (2, \dots, 2) \\ 1, & h(\alpha) = 2, \\ 0, & h(\alpha) \neq 2, \end{cases}$$

The function $h'(x_1, \dots, x_n) = g(m(x_1, \dots, x_n), 2)$ satisfies the property $h \sim h'$ by construction. In this way $h \in \mathcal{R}_\infty(\langle') \subseteq \mathcal{R}_\infty(\{\in, \mathcal{T}_{/\infty} \cap \mathcal{T}_\in\}) \subseteq \mathcal{R}_\infty(\{\{, \mathcal{T}_{/\infty} \cap \mathcal{T}_\in\})$. By the arbitrary function $h$ we have $T_2 \in \mathcal{R}_\infty(\{, \mathcal{T}_{/\infty} \cap \mathcal{T}_\in)$.

The lemma is proved.

Now it is possible to formulate the main result that follows from these lemmas

**Theorem 1** (Completeness) *There are five pre-complete classes in* $T_2$.

## 2.3 The Completeness Problem for the Operator $\mathcal{R}_\infty$

Let $M$ be a given set of functions from $P_3$. Denote the result of the closure of the set of functions $M$ with respect to operation of substitution and transition of the function $g$ to the equivalent function $f \sim g$ as $\mathcal{R}_\infty(\mathcal{M})$, where

$$f \sim g \Leftrightarrow \forall \mathbf{x}\, [\, (f(\mathbf{x}) = g(\mathbf{x})) \vee (f(\mathbf{x}), g(\mathbf{x}) \in \{0, 1\})\, ].$$

Consider classes: $T_{01}$ — class of functions preserving the set $\{0, 1\}$, $T_2$ — function class preserving two, and class $T_\sim$ (also $T_{\{01\},\{2\}}$ $(U(R))$ — function class, preserving the relation $\sim$ .

It is easy to see that with passing from the function $f$ to the function $g$ property of belonging to classes $T_2$, $T_{01}$, $T_\sim$ is preserved. In this way due to the fact that classes $T_2$, $T_{01}$, $T_\sim$ are precomplete with respect to the substitution, and completion does not add new functions, then the following lemma is obtained:

**Lemma 7** *Classes* $T_2$, $T_{01}$, $T_\sim$ *are* $\mathcal{R}_\infty$*-precomplete.*

**Lemma 8** *Let* $f \notin T_{01}$, *Then* $2 \in \mathcal{R}_\infty(\{)$.

***Proof*** It is easy to check that if $h(x) \notin T_{01}$ is a 1-place function, then $2 \in \mathcal{R}_\infty(\langle)$. Thus, if the function $g(x) = f(x, \dots, x)$, $g \notin T_{01}$, then the lemma is proved.

Let $g \in T_{01}$. By condition, there is a set $\boldsymbol{\alpha} = (a_1, \dots, a_n)$, $a_i \in \{0, 1\}$ such that $f(\boldsymbol{\alpha}) = 2$. Consider a function $f'$ such that

$$g'(x) = f(g_1(x), \dots, g_n(x))$$

where $g_i(g_i(0) = a_i)\&(g_i \sim g)$. notice, that $g' \in \mathcal{R}_\infty(\{)$ and $g'(0) = 2$, and then $2 \in \mathcal{R}_\infty(\}') \subseteq \mathcal{R}_\infty(\{)$.

The lemma is proved.

**Theorem 2** (Completeness) *There are three* $\mathcal{R}_\infty$*−pre-complete classes* $T_2$.

## 2.4 Conclusion

In this paper, the closure operators on the set of functions of three-valued logic, which are a strength of the usual substitution operator was considered. It was proved that the completeness problem for this operator has a solution; it is possible to recover the sublattice of closed classes in the general case of closure of functions with respect to the classical superposition operator, which will optimize possible production of chipsets for new functional circuits for transmission and data processing tasks. Also a brief overview of modern applications of three-valued logic models was given.

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